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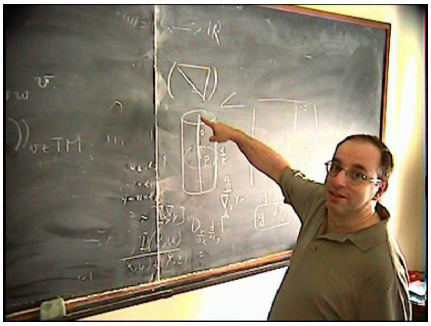
S. G. Chiossi, I. Dotti and A. Fino

IN MEMORIAM SERGIO CONSOLE



Sergio Console, professor at the University of Torino, tragically passed away in November 2013 at the age of 48. This note provides a summary of Sergio's work and our own way of remembering one of the most eclectic and remarkable persons we have ever known.

A personal view

We could not conceive of being around Sergio without being entranced by his humour and distinctive laughter that lit up a room. He got on with virtually anybody, and those who knew him understand at a deeper, subconscious level, what a marvellous individual he was. The abysmal sense of loss still lingers on and he will be forever missed, but we seek solace in the fact that he was part of our lives.

The authors of this note are lucky to have had many years of his friendship. They stand in, in Sergio's words, for three peoples he esteemed highly, and whose core values he embodied. First of all he was a proud *piemontéis* (Piedmont being the region surrounding Turin), and as such he had an inevitable penchant for the celebrated *toma*, a special local cheese that all his guests had to try. He was also a supporter of Torino FC, and used to drag friends and colleagues to pubs to watch its matches.
Secondly, he loved Argentina and the Argentine lifestyle unconditionally, to the point he considered Córdoba, which he visited several times, as his second home. He had even mentioned buying property there. The story of Sergio and Carlos Olmos lost in the Sierras de Córdoba in the winter of 1995 has become a matter of legend. Here is how *La Voz del Interior* newspaper reported that adventure:

Lost professors rescued. *The Police and the volunteers of the fire brigade of La Falda came to the aid of two university professors and the children of one of them who had gotten lost on 'la Banderita' mountain.*
On Sunday at 19.30, after a woman living in the town's centre heard distress calls, the rescue teams started ascending the mountain side and at a few hundred meters uphill found Sergio Console (30 years old), an Italian national resident at 327 Belgrano, Córdoba, Carlos Olmos (35), resident at 2935 Necochea, Córdoba, and the latter's two children Carlos (8) and Lucía (5).
The four were in good health and explained that they had begun the hike that afternoon, but on returning they had lost their way.

Socorrieron a profesores extraviados

Policías y bomberos voluntarios de La Falda socorrieron a dos profesores universitarios y a los hijos de uno de ellos, quienes se habían extraviado en el cerro "La Banderita".

A las 19.30 del domingo, una mujer domiciliada en la zona céntrica de la citada ciudad serrana escuchó voces requiriendo auxilio, por lo que los equipos de rescate iniciaron el rápido ascenso y a unos 100 metros de altura encontraron a Sergio Cónsole (30), de nacionalidad italiana y afincado en Belgrano 327; Carlos Enrique Olmos (35), domiciliado en Necochea 2935, ambos de esta Capital, y a los hijos de este último, de nombres Carlos (8) y Lucía (5).

Los cuatro estaban en buen estado de salud y comentaron que habían iniciado la caminata en horas de la tarde y durante el descenso equivocaron el camino correcto.

And finally, during the course of several stays Sergio absorbed many aspects of the German culture. He was keen to learn and speak the vernaculars, had a soft spot for wheat beers, was fond of pre-gentrified Berlin and loved the Neue Deutsche Welle films shown at the Goethe-Institut in Turin.
The earnest curiosity for different cultures and the readiness to soak up other customs and mores are also reflected in the fact that he was a fervid traveller. From his first big trip to Notre Dame, Indiana to Cologne, from Córdoba to Nicosia, from Berlin to Pocatello, he treasured the memories of all the places he visited and always retained bits of what he had been exposed to.

Another, important, side to Sergio was that in the last ten years or so he had become a dedicated jogger. He clocked up thousands of miles every year, and often managed to plan scientific visits to coincide with the local half-marathon. Such was his contagious enthusiasm that he got some colleagues into becoming ardent runners as well. This is not surprising given he was a rather outdoorsy kind of guy. Every free weekend he packed his running gear or his snowshoes, depending on the season, and headed for his beloved Alps. No one could keep up with him, no one ever will.

Scientific interests

Whoever discussed mathematics with Sergio remembers his love for the subject and how he cherished working with others. A quick glance at the list of publications (in the references) testimonies the importance of collaborations in his work. On the one hand this is a consequence of his need for sharing experiences, on the other it is evidence of

his easy-goingness. His many co-authors will attest he was a competent, inventive and generous mathematician.

Sergio worked in differential geometry (Riemannian and complex geometry, geometric structures, submanifold theory, homogeneous spaces) and algebraic topology (cohomology theories, fibrations, spectral sequences). The themes he was particularly concerned with are:

- submanifolds and holonomy (submanifolds of space forms, rank rigidity, isoparametric rank, normal holonomy, complex submanifolds)
- isometric actions of Lie groups (orbit spaces, polar actions, s-representations, locally homogeneous spaces, cohomogeneity)
- nilmanifolds and solvmanifolds (invariant geometric structures, deRham and Dolbeault cohomology, deformations of complex structures and stability)
- flat manifolds (issues of isospectrality, $\mathbb{Z}_2$-cohomology, spin structures)
- curvature invariants of Riemannian and Hermitian manifolds (Hermitian Singer number, scalar Weyl invariants).

Overview of the work

In the sketchy description that follows we have compressed a career spanning 25+ years. Inevitably we will overlook aspects that Sergio would consider relevant, for lack of understanding on our side.

Sergio studied at the University of Torino where he graduated with Alberto Collino. He was awarded a permanent lecturer post in Turin in 1990 despite never formally obtaining a PhD (an uncommon degree in Italy at the time). He was steered towards classical differential geometry mainly under the influence of the late Aristide Sanini, the mentor with whom Sergio tackled problems in submanifold theory related to harmonic maps. The first papers [1, 2, 4] classify surfaces with conformal second fundamental form II and their isotropic immersions in space forms. The affection for Sanini is clearly evident in [24] (contained in the volume [23]).

In 1992 Sergio visited the University of Notre Dame with the purpose of studying extrinsically homogeneous geometry with Gudlaugur Thorbergsson. He classified infinitesimally homogeneous Euclidean submanifolds [3] generalising work of Ferus and Singer. In 1994 he spent a follow-up year in Cologne as visiting professor, and worked with Thorbergsson on the geometric characterisation of orthogonal representations of compact Lie groups G, with an eye to s-representations (isotropy representations of symmetric spaces). These are orbit-equivalent to polar actions, and principal orbits of s-representation are isoparametric submanifolds. So they studied the relationships between polar representations, variationally complete representations, representations of class O_2 and taut representations [6]. In particular it was proved that if the orbits of G are all taut, or O_2, then G has four simple factors at most.

This line of thought was pursued with Anna Fino to characterise homogeneous Kählerian submanifolds of $\mathbb{CP}^n$ in representation-theoretical terms [5], and the link between maximal symmetric weights of an irreducible representation and the flag geometry of s-representations [11].
Along these lines, years later he wrote [29], once again with Fino and Thorbergsson, addressing Cartan's theory of isoparametric hypersurfaces in spheres with three principal curvatures. In it one finds the relationship between Hurwitz's composition algebras and the Cartan–Schouten theorem on flat metric connections with skew-symmetric torsion.

It was almost by chance that Sergio met Carlos Olmos (from Córdoba) in Florence in 1993, an event that was to mark the start of the long-lasting collaboration and friendship between the two. They undertook in [7] the investigation of isoparametric submanifolds and submanifolds with constant principal curvatures in relation to homogeneity issues, and more generally to situations where II is algebraically constant, following work of Heintze and Thorbergsson. They extended the notion of isoparametric rank (the maximal number of independent isoparametric and parallel local normal sections) to arbitrary submanifolds S of a space form. The main result is that a locally irreducible full submanifold S in a sphere with isoparametric rank at least 2 must have constant principal curvatures. Hence if the isoparametric rank is at least 1 then S is either an isoparametric hypersurface or the orbit of an s-representation. Many other 'high rank' theorems follow from this work, based on the observation that orbits of s-representations in submanifold geometry are the extrinsic relatives to Riemannian symmetric spaces in intrinsic geometry. For example, a submanifold of the sphere with parallel non-vanishing curvature and constant fundamental form must have constant principal curvatures.
A couple of trips to Córdoba led in 1998 to a paper on submanifolds with algebraically constant II. The result in [8] is that submanifolds in space forms with constant mean curvature and II of the type of a symmetric manifold are locally symmetric. From it one can recover, among other things, Cartan's celebrated classification of isoparametric hypersurfaces in spheres mentioned earlier.
A related aspect was addressed in [9], written with Carfagna D'Andrea, which shows that a submanifold in a space form with parallel higher second fundamental form is extrinsically 2-symmetric.

During the long time spent in Argentina over the years, Sergio also collaborated with Roberto Miatello and Juan Pablo Rossetti [25, 26]. Between one football match and the other they worked on topological and analytical aspects of compact flat quotients $M_\Gamma = \mathbb{R}^n/\Gamma$ by Bieberbach groups Γ. They considered in particular manifolds with diagonal holonomy, for which Γ is spanned by pairs (B,b) where $B = \mathrm{diag}(\pm 1,\dots,\pm 1) \in \mathrm{O}(n)$ and $b \in \frac{1}{2}\mathbb{Z}^n$. Computing the cohomology $\mathrm{H}^j(M_\Gamma, R) \cong \mathrm{H}^j(\Gamma, R)$ is easy when the coefficient ring is $\mathbb{R}$, but becomes harder for $R = \mathbb{Z}_2$. They determined the groups $\mathrm{H}^j(M_\Gamma, \mathbb{Z}_2)$ for $j = 1,2$ using the Lyndon–Hochschild–Serre spectral sequence, and gave an effective criterion for the non-vanishing of the Stiefel–Whitney class $w_2(M_\Gamma)$. The papers further contain the full $\mathbb{Z}_2$-cohomology of low-dimensional Hantzsche–Wendt manifolds, and exhibit examples of isospectral 5-ma-

nifolds with different H^2, and isospectral pairs of 4-manifolds, one spin and one not, with the same $\mathbb{Z}_2$-cohomology.

Sergio then became interested in finding complete sets of local metric invariants for Riemannian manifolds (M^n, g). A theorem of Singer states that if M is analytic and locally homogeneous, a finite number of derivatives of the curvature at one point solve the problem: in fact local homogeneity is equivalent to the existence, for any $p, q \in M$, of a linear isometry $h\colon T_pM \to T_qM$ such that $h^*(\nabla^i R)_p = (\nabla^i R)_q$ for any $i \leqslant n(n+1)/2$. He recast this fact with Lorenzo Nicolodi [10] in the Hermitian setting. They showed that for the theorem to hold on an almost Hermitian manifold (M, g, J) one needs to add the two derivatives $(\nabla J)_p, (\nabla^2 J)_p$ to the previous invariants. This in turn allows to employ algebraic infinitesimal models, and led to examples of locally homogeneous Hermitian manifolds that are not locally isometric to any globally homogeneous space.

The collaboration with Jürgen Berndt and Anna Fino [13] on the topology of real flag manifolds M appeared in 2001. The index number $\#_I(M)$ allowed to make use of the tools of symplectic topology (since the complexified flag $M^{\mathbb{C}}$ is a co-adjoint orbit) and Morse theory (height functions on M generically have non-degenerate critical points). By invoking standard Atiyah–Guillemin–Sternberg–Kostant–Duistermaat technology (Hamiltonian torus actions and convexity theorems) they proved that $\#_I(M)$ equals the smallest number of cells in a CW-structure of M.
Building on this, Sergio wrote the short note [15] with a symplectic flavour. In symmetric R-spaces M a maximal 2-set (a subset of mutually antipodal points, whose cardinality $\#_2(M)$ equals the sum of the $\mathbb{Z}_2$-Betti numbers) is given by the vertices of a certain convex polytope, image of M under a moment map. Using properties of isoparametric submanifolds and their focal manifolds, he proved that points in a maximal 2-set lying in adjacent chambers can be connected by half-geodesics mapped to lines by the moment map.

In 2001 Antonio Di Scala, Carlos Olmos and Sergio published a paper [14] relating the theory of submanifolds to Olmos's normal holonomy theorem, that is, the extrinsic version of de Rham's decomposition and Berger's Riemannian holonomy theorem. The paper was essentially an appetiser of the the landmark opus [16] that Sergio had been preparing for years with Berndt and Olmos. The book appeared in 2003 and was designed to be a thorough survey focussing on homogeneous and isoparametric submanifolds plus their generalisations. At its core lies the holonomy of the normal connection of a submanifold and the techniques ensuing from the normal holonomy theorem. These led to an improvement of the classical proofs, resulting in a more unified treatment. The text includes many exercises, and eventually discusses extensions of the theory to the broader class of symmetric spaces.
Similar ideas were taken up in [21, 27] to investigate complex submanifolds in $\mathbb{CP}^n$. The first article computes the normal holonomy of complex projective submanifolds M with parallel second fundamental form (corresponding to the unique complex orbit, in the projectivised space, of a Hermitian s-representation). The second paper is devoted to a Berger-type result for the normal holonomy of a full complete complex projective submanifold $M \subset \mathbb{CP}^n$: either the holonomy is transitive on the unit sphere

of the normal space, or M is the complex orbit of the isotropy representation of an irreducible Hermitian symmetric space of rank $\geqslant 3$. Moreover, the holonomy of complete irreducible complex submanifolds of $\mathbb{C}^n$ is transitive on the unit normal space, i.e. generic.

The two papers [20, 22] resumed the collaboration with Olmos on isometric actions on Riemannian manifolds (M^n, g), and curvature invariants. A scalar Weyl invariant is a function T whose components are polynomial functions in the curvature and its derivatives up to a given order, and must arise as a linear combinations of complete traces. By identifying Killing fields with parallel sections of the bundle $TM \oplus \bigwedge^2 M$, it was proven that the cohomogeneity of (M, g) with respect to the full isometry group of g coincides locally with the codimension of the foliation by regular level sets of the scalar Weyl invariants. In particular, this recasts the result of Prüfer and Tricerri–Vanhecke whereby M is locally homogeneous if and only if all scalar Weyl invariants of order $s \leqslant \binom{n}{2}$ are constant, and ties up nicely with [10]. The proof can be applied to pseudo-Riemannian manifolds and, as a matter of fact, torsion-free affine connections other than the Levi–Civita connection.

During an exchange programme set up with Simon Salamon, then at Oxford University, Sergio and Anna began the systematic study of explicit methods for computing the Dolbeault cohomology of a wide class of non-Kähler manifolds. Their interest lay in compact quotients of nilpotent Lie groups $M = N/\Gamma$ (nilmanifolds) equipped with left-invariant complex structures J, and the relation to the Chevalley–Eilenberg cohomology of the Lie algebra $\mathfrak{n}$ of N. The starting point is the inclusion $\bigwedge^{p,q}(\mathfrak{n}^{\mathbb{C}}) \hookrightarrow \bigwedge^{p,q}(M)$, which induces a graded injective morphism j on the respective $\bar{\partial}$-complexes. Using Borel spectral sequences they generalised a result of Kodaira and proved [12] the holomorphic version of Nomizu's theorem, i.e. that j becomes a full blown isomorphism $\mathrm{H}^{\mathrm{p,q}}_{\bar{\partial}}(N/\Gamma, J) \cong \mathrm{H}^{\mathrm{p,q}}(\mathfrak{n}^{\mathbb{C}})$ in certain situations, for example when J is Abelian ($\mathfrak{n}^{1,0} \subset \mathfrak{n}^{\mathbb{C}}$ is an Abelian subalgebra), or provided J is lattice-rational ($J\mathfrak{n}^{\mathbb{Q}} \subseteq \mathfrak{n}^{\mathbb{Q}}$). This theorem has become a standard and extremely practical tool. It has sparked a host of papers by other people, thus becoming very cited in the area. See the overview [19].
The same circle of ideas, developed with Anna and Yat Sun Poon in [18], shows that the above result can be extended to the cohomology with values in the holomorphic tangent sheaf Θ_M, to the effect that the cohomology ring $\mathrm{H}^*(M, \Theta_M)$ arises essentially from invariant differential forms, and the isomorphism j is stable under small deformations of invariant complex structures. In fact, the locally complete Kuranishi family of deformations of an Abelian complex structure on $M = N/\Gamma$ consists entirely of invariant complex structures. If $\dim_{\mathbb{R}} N = 6$ all Kuranishi deformations of an Abelian complex structure are nilpotent, but there are counterexamples in higher dimensions.

Evangelia Samiou collaborated with Anna and Sergio in a study of real two-step nilpotent Lie algebras of dimension up to 6 equipped with invariant Riemannian metrics in [17]. The moduli space $\mathcal{N}_6$ in dimension six is an explicit cone over a 4-dimensional contractible simplicial complex inside the variety of Lie algebras. Specifically, $\mathcal{N}_6 = \bigsqcup_{k=0}^{3} \mathcal{N}_{6,k}$, where the space $\mathcal{N}_{6,k}$ of Lie algebras with k-dimensional com-

mutator contains $\mathrm{Gr}(k, \Lambda^2\mathbb{R}^{n-k})/\mathrm{O}(n-k)$ as a strong deformation retract. For example, homotopically $\mathcal{N}_{6,0}$ is point, $\mathcal{N}_{6,1} \approx [0,1] \times \mathbb{R}^+$ and $\mathcal{N}_{6,3} \approx \mathrm{Sym}^2(\Lambda^2\mathbb{R}^3)^*/\mathrm{O}(3)$. Standard metric representatives for the seven isomorphism types, containing all deformations, were found.

The research programme for determining cohomology groups was pushed further to encompass the much broader and more complicated class of solvmanifolds G/Γ. It was tackled in a series of papers, the first of which, [28], with Anna Fino, considers the Mostow condition $\mathbb{T} \cdot \overline{\mathrm{Ad}\,\Gamma}^{\mathrm{alg}} = \overline{\mathrm{Ad}\,G}^{\mathrm{alg}}$, where $\mathbb{T}$ is some compact torus and the overline $\overline{}^{\mathrm{alg}}$ indicates algebraic closure. They show that under that assumption there exist a subgroup $\widetilde{\Gamma} \subseteq \Gamma$ of finite index and a simply connected normal subgroup $\widetilde{G} \subseteq \mathbb{T} \ltimes G$ with Lie algebra $\widetilde{\mathfrak{g}}$ such that $\mathrm{H}^*_{\mathrm{dR}}(\widetilde{G}/\widetilde{\Gamma}) \cong \mathrm{H}^*(\widetilde{\mathfrak{g}})$. This simplifies the proof of a theorem of Guan Zhuang-dan and can be adapted to compute the Betti numbers of compact solvmanifolds failing Mostow's condition.
Since Nomizu's theorem is no longer valid for solvmanifolds, Sergio and Anna joined forces with Hisashi Kasuya [31] to construct a new Lie algebra $\overline{\mathfrak{g}}$ out of $\mathfrak{g}$ satisfying the property, both in the de Rham and Dolbeault situations.
Together with Gabriela Ovando and Mauro Subils [30], Sergio considered compact quotients of the oscillator group $G \cong \mathbb{R} \times \mathcal{H}^3$. Taking Heisenberg subgroups $\Gamma_k \subset \mathcal{H}^3$ they built three families of lattices $\Lambda_{k,0}$, $\Lambda_{k,\pi}$, $\Lambda_{k,\pi/2}$ in G and determined the topology of the corresponding quotients $M_{k,\theta} = G/\Lambda_{k,\theta}$. Only the first solvmanifolds ($\theta = 0$) have the same Betti numbers of the Kodaira–Thurston model, but their cohomology disagrees with the invariant one. For the other two families ($\theta = \pi, \pi/2$) Nomizu's theorem holds whereas the Mostow condition does not. These examples may be distinguished by means of their invariant geometry: for instance, the spaces $M_{k,0}$ all inherit symplectic structures from G, none of which is invariant though. In contrast, the cohomology of $M_{k,\pi}$, $M_{k,\pi/2}$ prevents them from being symplectic.
The paper [32] is based on Maura Macrì's doctoral thesis, that Sergio supervised: it constructs explicit lattices in six-dimensional, non-completely solvable almost Abelian Lie groups violating Mostow's condition, and studies the ensuing geometry and topology.

In the summer of 2012 Sergio travelled to Pocatello (in Idaho) and Vancouver, on what was to be his last scientific trip.

A gifted, dedicated and humble worker

Next to the scholar was a natural born teacher. By imparting a serious attitude towards work, infusing students with an overall vision and passing on the gusto of doing mathematics, he shaped the minds of many. He accumulated 34 graduate students in mathematics. Even those who came across him in passing – typically students of service courses, with less than an inkling for mathematical abstraction – remember him with profound affection, even after years, as 'the' inspirational teacher one would want to learn from. See the massive emotional response left in the book of condolences, available online at

http://teachingdm.unito.it/console/.

For a number of years Sergio had also engaged in guest lecture courses outside Turin. His colleague and running mate Francesca Ferrara remembers fondly the linear algebra & geometry course they taught together in English at the University of Biella.

The astonishing, almost total absence of doctoral students (he supervised one PhD thesis only) is yet another manifestation of his humbleness. In his own words he was 'not up to the job', which of course we all knew was untrue. Because he never was the flamboyant or pushy type, amid an ocean of sharks, his unassertiveness and good nature resulted in a long stint as lecturer. He obtained the Italian national habilitation for associate professorship in 2013.

What Sergio lacked in ambition he made up for in commitment. His closest friends in the Maths' Department – Andrea Mori, Elsa Abbena, Ernesto Buzano, Francesca Ferrara, Luigi Vezzoni, Sergio Garbiero – always expressed great admiration for the resolve and the energy he put in all administrative duties. He was in charge with Fino of long-running and very profitable exchange programs with FaMAF/UNC in Argentina and Oxford University. In Turin he sat on a number of managing boards: Steering Committee of the Department of Mathematics (1994–97), Teaching Committee for the Faculties of Natural Sciences (2001–06) and Chemistry (2005–10), Committee for Tutorial Activities in Mathematics (2006–11). A particularly vexing job he took on was to coordinate the dramatic architectural restructuring and refurbishing of the century-old department building, at a time (2011–13) when safety regulations had been updated and a colossal intervention was thus required. Always seeing the bright side of things, he used to say half-jokingly that tearing down walls to open new corridors, blocking existing passages and moving staircases was the perfect punishment for someone like him who loved topology.

Let us mention, in closing, that he served as member of the Scientific Committee (1997–2008) and as executive secretary (2003–08) of the *Seminario Matematico dell'Università e del Politecnico di Torino*, publisher of this journal.

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Anna Fino
Dipartimento di Matematica 'G.Peano', Università di Torino
via Carlo Alberto, 10, 10123 Torino (IT)
e-mail: annamaria.fino@unito.it

Isabel Dotti
Facultad de Matemática, Astronomía y Física, Universidad Nacional de Córdoba

Medina Allende s/n, Ciudad Universitaria, 5000 - Córdoba (AR)
e-mail: `idotti@famaf.unc.edu.ar`

Simon G. Chiossi
Departamento de Matemática Aplicada, Instituto de Matemática e Estatística,
Universidade Federal Fluminense
Rua Mário Santos Braga s/n, 24020-140 Niterói/RJ (BR)
e-mail: `simongc@id.uff.br`

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