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ON MULTIPLICATIVE (GENERALIZED)-SKEW DERIVATIONS OVER SEMIPRIME RINGS

Abstract. Let R be a semiprime ring with center $Z(R)$ and let F be a multiplicative (generalized)-skew derivation of R associated with an automorphism α. In the present paper, we study the following situations:$(i)F(xy)\pm\alpha(xy)=0$, $(ii)F(xy)\pm\alpha(yx)=0$, $(iii)F(x)F(y)\pm\alpha(x)y=0$, $(iv)F(xy)\pm\alpha(x)y\in Z(R)$ and $(v)F(x)F(y)\pm\alpha(x)y\in Z(R)$ for all x,y in some appropriate subset of R.

1. Introduction

Throughout this paper, R will represent some associative ring with center $Z(R)$. For any $x,y\in R$, the symbol $[x,y]$ stands for the commutator $xy-yx$ while $x\circ y$ represents the anti-commutator $xy+yx$. Recall that a ring R is prime if for any $a,b\in R$, $aRb=\{0\}$ implies that $a=0$ or $b=0$, and R is semiprime if $aRa=\{0\}$ implies $a=0$. An additive mapping $d:R\longrightarrow R$ is called a derivation if $d(xy)=d(x)y+xd(y)$ holds for all $x,y\in R$. In particular, for a fixed $a\in R$, the mapping $I_a:R\longrightarrow R$ given by $I_a(x)=[x,a]$ is a derivation called an inner derivation. A mapping $f:R\longrightarrow R$ is called centralizing on a non-empty subset S of R if $[f(x),x]\in Z(R)$ for all $x\in S$ and is called commuting on S if $[f(x),x]=0$ for all $x\in S$.

An additive mapping $F:R\longrightarrow R$ is called a generalized inner derivation if $F(x)=ax+xb$ for fixed $a,b\in R$. For such a mapping F, it is easy to see that

$$F(xy)=F(x)y+x[y,b]=F(x)y+xI_b(y) \text{ for all } x,y\in R.$$

This observation leads to the following definition, given in [10]; an additive mapping $F:R\longrightarrow R$ is called a generalized derivation with associated derivation d if

$$F(xy)=F(x)y+xd(y) \text{ holds for all } x,y\in R.$$

Familiar examples of generalized derivations are derivations and generalized inner derivations and the latter includes left multiplier i.e., an additive map $F:R\longrightarrow R$ satisfying $F(xy)=F(x)y$ for all $x,y\in R$. Since the sum of two generalized derivations is a generalized derivation, every map of the form $F(x)=cx+d(x)$, where c is fixed element of R and d a derivation of R, is a generalized derivation; and if R has 1, all generalized derivations have this form. An additive mapping $d:R\rightarrow R$ is said to be skew derivation associated with an automorphism α if $d(xy)=d(x)y+\alpha(x)d(y)$ holds for all $x,y\in R$. Basic examples of skew derivations are usual derivations and the mapping $\alpha-id$, where id denotes the identical mapping of R. The notion of skew derivation

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has been extended to generalized skew derivation. An additive mapping $F : R \to R$ is said to be a (right) generalized skew derivation associated with an automorphism α if there exists a skew derivation $D : R \to R$ associated with an automorphism α such that $F(xy) = F(x)y + \alpha(x)d(y)$ holds for all $x, y \in R$. A mapping $F : R \longrightarrow R$ satisfying $F(xy) = F(x)\alpha(y)$ for all $x, y \in R$ is called a multiplicative α-left centralizer of R.

Let S be a nonempty subset of a ring R. The mapping $F : R \longrightarrow R$ is said to be a homomorphism (anti-homomorphism) acting on S if $F(xy) = F(x)F(y)$ holds for all $x, y \in S$ (respectively $F(xy) = F(y)F(x)$ holds for all $x, y \in S$). In [8] Bell and Kappe proved that if a derivation d of a prime ring R acts as a homomorphism or as an anti-homomorphism on a nonzero right ideal of R, then $d = 0$ on R. Ali et al. [2] proved similar results in the Lie ideal case. In fact, they proved that if R is a 2-torsion free prime ring admitting a derivation d which acts either as a homomorphism or as an anti-homomorphism on a nonzero square closed Lie ideal L of R, then either $d = 0$ or $L \subseteq Z(R)$. Also in [21], Rehman generalized this results as follows;

THEOREM 1. *Let R be a 2-torsion free prime ring and I be a non zero ideal of R. Suppose $F : R \longrightarrow R$ is a nonzero generalized derivation with associated derivation d.*

(i) If F acts as a homomorphism on I and if $d \neq 0$, then R is commutative.,

(ii) If F acts as an anti-homomorphism on I and if $d \neq 0$, then R is commutative.

Recently, Dhara [14], has studied the situations, when a generalized derivation F of a semi prime ring R acts as a homomorphism or as an anti-homomorphism on a non zero left ideal of R. Ashraf et al. [5] proved that the prime ring R is commutative if R satisfies any one of the following conditions: $(i)F(xy) - xy \in Z(R)$ for all $x, y \in I$, $(ii)F(xy) + xy \in Z(R)$ for all $x, y \in I$, $(iii)F(xy) - yx \in Z(R)$ for all $x, y \in I$, $(iv)F(xy) + yx \in Z(R)$ for all $x, y \in I$, $(v)F(x)F(y) - xy \in Z(R)$ for all $x, y \in I$, $(vi)F(x)F(y) + xy \in Z(R)$ for all $x, y \in I$, where F is a generalized derivation of R with nonzero derivation d and I a nonzero two sided ideal of R. Motivated by these results, Dhara et al. [17] studied the situations: $(i)F(xy) - F(x)F(y) \in Z(R)$, $(ii)F(xy) + F(x)F(y) \in Z(R)$, $(iii)F(xy) - F(y)F(x) \in Z(R)$ and $(iv)F(xy) + F(y)F(x) \in Z(R)$ for all x, y in some suitable subset of R. Again Dhara et al. [16] generalized the results of Ashraf et al. [5] with multiplicative (generalized) derivation.

A mapping $d : R \longrightarrow R$ (not necessarily additive)satisfying $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$ is called a multiplicative derivation of R. A mapping $d : R \longrightarrow R$ (not necessarily additive)is called a multiplicative α-derivation associated with a map $\alpha : R \longrightarrow R$ such that $d(xy) = d(x)\alpha(y) + xd(y)$ for all $x, y \in R$. This concept was given by Daif [11]. Further, such maps were studied by Goldman and Semrl [18]. This notion was generalized by Daif and Tammam El- Sayiad [13] as follows: A mapping $F : R \longrightarrow R$ is called a multiplicative generalized derivation if there exists a derivation $d : R \longrightarrow R$ such that $F(xy) = F(x)y + xd(y)$ for all $x, y \in R$. Recently, Dhara and Ali [16], generalized the concept of multiplicative generalized derivation to multiplicative (generalized)-derivation as follows: A mapping $F : R \longrightarrow R$ (not necessarily additive)is called a multiplicative (generalized)- derivation if $F(xy) = F(x)y + xg(y)$ for all $x, y \in R$, where g is any mapping (not necessarily a derivation nor an additive map) For example of such maps we refer to [16]. In the present paper we generalize the concept of multiplicative (generalized)- derivation to multiplicative (generalized)-skew derivation. A mapping $F : R \longrightarrow R$ (not necessarily additive)is called a multiplicative (generalized)- skew derivation if $F(xy) = F(x)y + \alpha(x)g(y) = F(x)\alpha(y) + xg(y))$ for all $x, y \in R$, where $g : R \longrightarrow R$ is any mapping (not necessarily a skew derivation nor an additive map) and $\alpha : R \longrightarrow R$ is an automorphism of R. The following example shows the existence of such maps.

EXAMPLE 1. Let $R = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \mid a,b,c \in Z \right\}$. If we define the mappings $\alpha, F, g : R \longrightarrow R$ such that $\alpha \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} a & -b \\ 0 & c \end{pmatrix}$, $F \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 0 & a+c \\ 0 & 0 \end{pmatrix}$ and $g \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 0 & a-c \\ 0 & 0 \end{pmatrix}$, then it can be easily verified that F is a multiplicative (generalized)-skew derivation. i.e., $F(xy) = F(x)\alpha(y) + xg(y) = F(x)y + \alpha(x)g(y)$ for all $x, y \in R$.

In the present paper, we study the following conditions: $(i) F(xy) \pm \alpha(xy) = 0$, $(ii) F(xy) \pm \alpha(yx) = 0$, $(iii) F(x)F(y) \pm \alpha(x)y = 0$, $(iv) F(xy) \pm \alpha(x)y \in Z(R)$ and $(v) F(x)F(y) \pm \alpha(x)y \in Z(R)$ for all x, y in some appropriate subset of R where $F : R \longrightarrow R$ is a multiplicative (generalized)- skew derivation of R associated with a map $g : R \longrightarrow R$ (not necessarily an additive map) and an automorphism α of R.

2. Main Results

THEOREM 2. *Let R be a semiprime ring, I a nonzero left ideal of R and $F : R \longrightarrow R$ be a multiplicative (generalized)- skew derivation associated with an automorphism α and a map $g : R \longrightarrow R$ satisfying $F(xy) \pm \alpha(xy) = 0$, for all $x, y \in I$. Then $Ig(I) = \{0\}$ and F is a multiplicative α-left centralizer on I.*

Proof. From the assumption given, let us consider $F(xy) - \alpha(xy) = 0$ for all $x, y \in I$. Replacing y by yz, where $z \in I$, we get $F(xyz) - \alpha(xyz) = 0$ which can be expanded as $F(xy)\alpha(z) + xyg(z) - \alpha(x)\alpha(y)\alpha(z) = 0$. This yields that $(F(xy) - \alpha(xy))\alpha(z) + xyg(z) = 0$ and by our hypothesis, this reduces to $xyg(z) = 0$ for all $x, y, z \in I$. Replacing y by $g(z)rx$, where $r \in R$, we obtain $xg(z)rxg(z) = 0$ for all $r \in R$. i.e., $xg(z)Rxg(z) = \{0\}$ for all $x, z \in I$. The semiprimeness of R implies that $xg(z) = 0$ for all $x, z \in I$ and hence $Ig(I) = \{0\}$. Thus for any $x, y \in I$, we have $F(xy) = F(x)\alpha(y) + xg(y) = F(x)\alpha(y)$ as desired.

In a similar manner, we can prove the same conclusions when $F(xy)+\alpha(xy)=0$, for all $x,y\in I$. □

COROLLARY 1. *Let R be a semiprime ring, I a nonzero left ideal of of R and $F: R\longrightarrow R$ be a multiplicative (generalized)- derivation associated with the map $g: R\longrightarrow R$ satisfying $F(xy)\pm xy=0$, for all $x,y\in I$. Then $Ig(I)=\{0\}$, F is a multiplicative left centralizer on I and commuting on I.*

Proof. By taking α the identity map of R, Theorem 2 implies that $Ig(I)=\{0\}$ and F is a multiplicative left centralizer on I. Using the given hypothesis, we get $(F(x)\pm x)y=0$ for all $x,y\in I$. By Theorem 2.1 of [16], we get $[F(x),x]=0$ for all $x\in I$, which implies that F is commuting on I. □

THEOREM 3. *Let R be a semiprime ring, I a nonzero left ideal of R and $F: R\longrightarrow R$ be a multiplicative (generalized)- skew derivation associated with an automorphism α and a map $g: R\longrightarrow R$ satisfying $F(xy)\pm\alpha(yx)=0$, for all $x,y\in I$. Then $Ig(I)=\{0\}$ and F is a multiplicative α-left centralizer of I.*

Proof. From the assumption given, let us consider $F(xy)-\alpha(yx)=0$ for all $x,y\in I$. Replacing y by yz, where $z\in R$, we get $F(xyz)-\alpha(yzx)=0$ which implies that $F(xy)\alpha(z)+xyg(z)-\alpha(y)\alpha(z)\alpha(x)=0$ which can be written as $(F(xy)-\alpha(y)\alpha(x))\alpha(z)+\alpha(y)\alpha(x)\alpha(z)+xyg(z)-\alpha(y)\alpha(z)\alpha(x)=0$. By using the hypothesis, we get $xyg(z)+\alpha(y)[\alpha(x),\alpha(z)]=0$ for all $x\in I$. In particular, we get $xyg(x)=0$ for all $x,y\in I$. Now replacing y by $g(x)ry$, where $r\in R$ in the last relation, we have $xg(x)Rxg(x)=\{0\}$ for all $x\in I$. The semiprimeness of R implies that $xg(x)=0$ for all $x\in I$ and hence $Ig(I)=\{0\}$. Thus for any $x,y\in I$, we have $F(xy)=F(x)\alpha(y)+xg(y)=F(x)\alpha(y)$ as desired.

In a similar manner, we can prove the same conclusions when $F(xy)+\alpha(yx)=0$, for all $x,y\in I$. □

COROLLARY 2. *Let R be a semiprime ring, I a nonzero left ideal of of R and $F: R\longrightarrow R$ be a multiplicative (generalized)- derivation associated with the map $g: R\longrightarrow R$ satisfying $F(xy)\pm yx=0$, for all $x,y\in I$. Then $Ig(I)=\{0\}$, F is a multiplicative left centralizer on I and F is commuting on I.*

Proof. By taking α the identity map of R, Theorem 3 implies that $Ig(I)=\{0\}$ and F is a multiplicative left centralizer on I. Using the given hypothesis, we get $F(x)y\pm yx=0$ for all $x,y\in I$. By using the same technique used in the proof of Theorem 2.3 of [16], we get $[F(x),x]=0$ for all $x\in I$, which implies that F is commuting on I. □

THEOREM 4. *Let R be a semiprime ring and $F: R\longrightarrow R$ be a multiplicative (generalized)- skew derivation associated with an automorphism α and a map $g: R\longrightarrow R$ satisfying $F(x)F(y)\pm\alpha(xy)=0$, for all $x,y\in R$. Then $Rg(R)=\{0\}$, F is a multiplicative α-left centralizer of R and $[F(x),\alpha(x)]=0$ for all $x\in R$.*

Proof. From the assumption given, let us consider

$$F(x)F(y) - \alpha(xy) = 0 \tag{2.1}$$

for all $x, y \in R$. Replacing y by yz in (2.1), where $z \in R$, we get $F(x)F(yz) - \alpha(xyz) = 0$ which implies that $F(x)\{F(y)\alpha(z) + yg(z)\} - \alpha(x)\alpha(y)\alpha(z) = 0$. Using relation (2.1), this reduces to $F(x)yg(z) = 0$, for all $x, y \in R$. Replacing x by wx in the last relation and then using it, we get $wg(x)yg(z) = 0$ for all $x, y, z, w \in R$. Replacing y by ry, the latter relation yields that $xg(x)Rxg(x) = \{0\}$ for all $x \in R$. The semiprimeness of R implies that $xg(x) = 0$ for all $x \in R$ and hence $Rg(R) = \{0\}$. Thus for any $x, y \in R$, we have $F(xy) = F(x)\alpha(y) + xg(y) = F(x)\alpha(y)$ as desired.
Replacing x by xy in (2.1) and using the fact $Rg(R) = \{0\}$, we get

$$F(x)\alpha(y)F(y) - \alpha(x)\alpha(y^2) = 0 \tag{2.2}$$

for all $x, y \in R$. Also right multiplying (2.1) by $\alpha(y)$, we get

$$F(x)F(y)\alpha(y) - \alpha(x)\alpha(y^2) = 0 \tag{2.3}$$

for all $x, y \in R$. Substracting (2.3) from (2.2), we have $F(x)[F(y), \alpha(y)] = 0$ for all $x, y \in R$. Replacing x by xz in the last relation and using the fact that $Rg(R) = \{0\}$, we have $F(x)\alpha(z)[F(y), \alpha(y)] = 0$ for all $x, y, z \in R$. Since α is an automorphism, replacing z by $\alpha^{-1}(z)$, the latter relation yields that

$$F(x)z[F(y), \alpha(y)] = 0 \tag{2.4}$$

for all $x, y, z \in R$. Again replacing z by $\alpha(x)z$ in (2.4), we get

$$F(x)\alpha(x)z[F(y), \alpha(y)] = 0 \tag{2.5}$$

for all $x, y, z \in R$. Left multiplying (2.4) by $\alpha(x)$, we have

$$\alpha(x)F(x)z[F(y), \alpha(y)] = 0 \tag{2.6}$$

for all $x, y, z \in R$. Combining (2.5)and (2.6), we find that $[F(x), \alpha(x)]z[F(y), \alpha(y)] = 0$ for all $x, y, z \in R$. In particular, we have $[F(x), \alpha(x)]R[F(x), \alpha(x)] = \{0\}$ for all $x \in R$. Semiprimeness of R implies that $[F(x), \alpha(x)] = 0$ for all $x \in R$.

In a simillar manner, we can prove the same conclusions when $F(x)F(y) + \alpha(xy) = 0$, for all $x, y \in R$. □

COROLLARY 3. *Let R be a semiprime ring and $F : R \longrightarrow R$ be a multiplicative (generalized)-derivation associated with the map $g : R \longrightarrow R$ satisfying $F(x)F(y) \pm xy = 0$, for all $x, y \in R$. Then F is a multiplicative left centralizer of R and F is commuting on R .*

THEOREM 5. *Let R be a semiprime ring and $F : R \longrightarrow R$ be a multiplicative (generalized)- skew derivation associated with an automorphism α and a map $g : R \longrightarrow R$ satisfying $F(xy) \pm \alpha(x)y \in Z(R)$, for all $x, y \in R$. Then $[g(x), x] = 0$, for all $x \in R$.*

Proof. From the assumption given, let us consider

$$F(xy) - \alpha(x)y \in Z(R) \tag{2.7}$$

for all $x, y \in R$. Replacing y by yz, where $z \in R$, we get $\{F(xy) - \alpha(xy)\}z + \alpha(xy)g(z) \in Z(R)$ for all $x, y, z \in R$. In view of (2.7), the latter relation reduces to as $[\alpha(xy)g(z), z] = 0$ for all $x, y, z \in R$. Replacing x by $\alpha^{-1}(x)$ and y by $\alpha^{-1}(y)$ in the last relation yields that $[xyg(z), z] = 0$ for all $x, y, z \in R$. Replacing x by rx where $r \in R$, we obtain $0 = [rxyg(z), z] = r[xyg(z), z] + [r, z]xyg(z) = [r, z]xyg(z)$ for all $x, y, z, r \in R$. By taking $x = g(z)x$, the latter relation gives that $[r, z]g(z)xyg(z) = 0$ for all $x, y, z \in R$. This can be written as $[r, z]g(z)Rxyg(z) = \{0\}$, for all $x, y, z, r \in R$. Interchanging in x and y and then subtracting one from the other, we get $[r, z]g(z)R[x, y]g(z) = \{0\}$, for all $x, y, z, r \in R$. In particular,$[x, z]g(z)R[x, z]g(z) = \{0\}$, for all $x, z \in R$. The semiprimeness of R yields that $[x, z]g(z) = 0$ for all $x, z \in R$. Right multiplying this by z and replacing x by xz in the latter relation, we obtain $[x, z]g(z)z = 0$ and $[x, z]zg(z) = 0$ for all $x, z \in R$. Combining the last two relations, we obtain $[x, z][g(z), z] = 0$ for all $x, z \in R$. Replacing x by $g(z)x$ in the last relation and then using it, we get $[g(z), z]x[g(z), z] = 0$ for all $x, z \in R$. i.e., $[g(z), z]R[g(z), z] = \{0\}$ for all $z \in R$. The semiprimeness of R yields that $[g(x), x] = 0$ for all $x \in R$.

In a simillar manner, we can prove the same conclusions when $F(xy) + \alpha(x)y = 0$, for all $x, y \in R$. □

COROLLARY 4. *Let R be a semiprime ring and $F : R \longrightarrow R$ be a multiplicative (generalized)-derivation associated with the map $g : R \longrightarrow R$ satisfying $F(x)F(y) \pm xy = 0$, for all $x, y \in R$, then g is commuting on R.*

THEOREM 6. *Let R be a semiprime ring and $F : R \longrightarrow R$ be a multiplicative (generalized)- skew derivation associated with an automorphism α and a map $g : R \longrightarrow R$ satisfying $F(x)F(y) \pm \alpha(x)y \in Z(R)$, for all $x, y \in R$, then $[g(x), x] = 0$, for all $x \in R$.*

Proof. From the assumption, let us take

$$F(x)F(y) - \alpha(x)y \in Z(R) \tag{2.8}$$

for all $x, y \in R$. Replacing y by yz, where $z \in R$, we get $F(x)\{F(y)z + \alpha(y)g(z) - \alpha(x)yz \in Z(R)$ which implies after using (2.8) that $[F(x)\alpha(y)g(z), z] = 0$ for all $x, y, z \in R$. Replacing y by $\alpha^{-1}(y)$ in the latter relation, we get

$$[F(x)yg(z), z] = 0 \tag{2.9}$$

for all $x, y, z \in R$. Replacing y by zy in (2.9), we get that

$$[F(x)zyg(z), z] = 0 \tag{2.10}$$

for all $x, y, z \in R$. Replacing x by xz in (2.9), we obtain $\{[F(x)z + \alpha(x)g(z)\}yg(z), z] = 0$ for all $x, y, z \in R$. Using (2.10), last relation reduces to $[\alpha(x)g(z)yg(z), z] = 0$ for all

$x,y,z \in R$. Now replacing x by $\alpha^{-1}(x)$, implies that $[xg(z)yg(z),z] = 0$. Replacing x by $g(z)x$ in the latter relation and then using it again, we obtain $[g(z),z]xg(z)yg(z) = 0$ for all $x,y,z \in R$. On some appropriate substitutions on x and y in the last relation, we get $[g(z),z]x[g(z),z]y[g(z),z] = 0$, for all $x,y,z \in R$. That is $(R[g(z),z])^3 = \{0\}$ for $z \in R$. Since R is semiprime, we find that $R[g(z),z] = \{0\}$ for $z \in R$. Then semiprimeness of R implies that $[g(x),x] = 0$ for all $x \in R$ as desired.

In a similar manner, we can prove the same conclusions when $F(x)F(y)+\alpha(x)y \in Z(R)$, for all $x,y \in R$.

□

COROLLARY 5. *Let R be a semiprime ring and $F : R \longrightarrow R$ be a multiplicative (generalized)- derivation associated with the map $g : R \longrightarrow R$ satisfying $F(x)F(y) \pm xy \in Z(R)$, for all $x,y \in R$. Then g is commuting on R.*

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