Rend. Sem. Mat. Univ. Politec. Torino
Vol. 71, 3–4 (2013), 421 – 461

L. Giacardi

THE ITALIAN SCHOOL OF ALGEBRAIC GEOMETRY AND THE TEACHING OF MATHEMATICS IN SECONDARY SCHOOLS: MOTIVATIONS, ASSUMPTIONS AND STRATEGIES

Abstract. In this paper I intend to illustrate the reasons which led some members of the Italian School of algebraic geometry – in particular, Corrado Segre (1863-1924), Guido Castelnuovo (1865-1952), Federigo Enriques (1871-1946) and Francesco Severi (1879-1961) – to become so concerned with problems pertaining to mathematics teaching; describe the epistemological vision which inspired them; discuss the various ways in which this commitment manifested itself (school legislation, teacher training, textbooks, publishing initiatives, university lectures, etc.); make evident the influence of the reform movements abroad, particularly that of Klein; finally, show how, in this respect as well, Italian geometers projected an unquestionable image of a "School".

1. Introduction

I am very pleased to take part in this conference dedicated to Alberto Conte in celebration of his seventieth birthday. The reason I was invited to a such a strictly gathering of mathematicians is due to the fact that Alberto Conte, along with his research work in the field of algebraic geometry, has always cultivated an interest in history of mathematics, both that of his specific area of study and that more generally related to Turin's rich scientific tradition. Further, with great sensitivity he has encouraged and supported initiatives aimed at bringing that tradition to broader attention.

I limit myself to recalling that in 1987, on the occasion of the 13th Congress of the Italian Mathematical Union, he encouraged Silvia Roero and myself to explore the patrimony of books and manuscripts housed in the libraries of Turin in order to highlight, by means of an exhibition and a catalogue, the wealth of mathematical studies in Piedmont and connections with international scientific research.[1] Noteworthy among his various works in the history of mathematics is the critical edition of the letters of Federigo Enriques to Guido Castelnuovo, entitled *Riposte armonie*, edited together with Umberto Bottazzini and Paola Gario[2], an extraordinary document testifying to the scientific and human fellowship of the two mathematicians. But I would also like mention one of the research projects we carried out together, regarding the

This research was carried out as a part of the project PRIN 2009, *Mathematical Schools in North Italy in* 18^{th} *and* 20^{th} *Centuries and Relationships with the International Community*, University of Turin.

[1] The Catalogue of the exhibition *Bibliotheca Mathematica. Documenti per la storia della matematica nelle biblioteche torinesi*, L. Giacardi and C. S. Roero, Eds. (Torino, Allemandi, 1987) was given as a gift to participants at the congress and has become a point of departure for many successive studies on mathematics in Piedmont.

[2] Bottazzini, Conte, Gario 1996.

physical-mathematical sciences in Turin during the period of French domination at the beginning of the nineteenth century, and to cite the contribution made by Alberto Conte to the reconstruction, based on in-depth examination of archives, of the large number of young students from Piedmont, including Giovanni Plana, who attended the École Politechnique, and of the kind of studies that they carried out.[3]
The topic I have chosen for this conference conjoins both of Alberto Conte's interests: algebraic geometry and the history of mathematics.

As it is well known, the Italian School of algebraic geometry was born in Turin at the end of the nineteenth century, under the guidance of Corrado Segre. It soon brought forth such significant results that it became a leading light (*führende Stellung*) on an international level, as F. Meyer and H. Mohrmann affirm in the *Encyclopädie der mathematischen Wissenschaften*.[4] Segre inspired an atmosphere of work characterised by highly prolific, enthusiastic, and frenetic activity, which Guido Castelnuovo, remembering his years in Turin, would refer to as "Turin's geometric orgies". The mathematicians involved were gifted students preparing their degree theses with Segre, such as Gino Fano (1892), Beppo Levi (1896), Alberto Tanturri (1899), Francesco Severi (1900), Giovanni Zeno Giambelli (1901), Alessandro Terracini (1911), and Eugenio Togliatti (1912). A number of newly graduated students from Italy and abroad were also drawn to Turin by Segre's fame. Amongst these, the most famous were Castelnuovo (1887-1891), Federico Amodeo (1890-91), Federigo Enriques (November 1992, November 1893-January 1894), Gaetano Scorza (1899-1900), the English couple William H. Young and Grace Chisholm Young (1898-99), and, from the United States, Julian Coolidge (1903-04), and C. H. Sisam (1908-09).

The great significance of the scientific results obtained by the School has led historians of mathematics to overlook, or at best to attach only secondary importance to the issues related to mathematics teaching that would occupy many of its members, including Segre himself, throughout their lives. It is only recently that various studies – which I will cite presently – have begun to explore this aspect as well of the activities of Italian geometers.
Broadly speaking, they shared a vision of mathematics teaching which derived, on the one hand, from their contacts with Felix Klein and his important movement of reforming the teaching of mathematics in secondary and higher education, on the other, from the way in which the authors themselves conceived of advanced scientific research. If, however, we look more closely at their contributions, from these common roots there emerge diverse motivations underlying their involvement with and different approaches to the problem.

The period of reference here goes from the final years of the nineteenth century through the first half of the twentieth century, and the institutional context which frames their commitment to education is still characterised, in the first two decades, by the Casati Law (1859), with attempts at reform either unsuccessful, or carried out

[3] A. Conte, L. Giacardi, *La matematica a Torino, in Ville de Turin 1798-1814*, Città di Torino, Torino, pp. 281-329.

[4] W. Fr. Meyer and H. Mohrmann, *Vorrede zum dritten Banden*, Encyklopädie der mathematischen Wissenschaften, III.I.1, 1907-1910, p. VI.

only in part. Such was the fate of the important reform project proposed by the Royal Commission (1909) to which Giovanni Vailati contributed. The project was never approved, although some of Vailati's proposals, as we shall see, were adopted by Castelnuovo in designing the programs for the *liceo moderno* (1913). The rise of Fascism and the Gentile Reform (1923) nullified any attempt at renovation in the area of science, notwithstanding the battle (carried out by some mathematicians such as Castelnuovo and Enriques) to restore dignity to mathematics, which had been strongly devalued by that reform.[5]

2. The spread in Italy of Felix Klein's ideas on education

It is well known that Felix Klein (1849-1925) always combined advanced-level research with serious attention to organizational and didactic problems pertaining to mathematics teaching at both the secondary and the advanced level.[6] Klein's interest in such problems can be dated back to 1872 when he obtained the professorship at the Erlangen University: in his *Antrittsrede* he described his own conception of mathematics, outlining a vision of teaching that would mature over the next thirty years. In the mid-1890s he became concerned with teacher training, organising a series of seminars for secondary teachers and publishing the *Vorträge über Ausgewählte Fragen der Elementargeometrie* (1895), with the aim of opposing the trend, then prevailing, towards approaches to mathematics that were too formal and abstract, and to resist overspecialization. Further, from the first he believed that the whole sector of mathematics teaching, from its very beginnings at elementary school right up to the most advanced research level, should be regarded as an organised whole:

> "It grew ever clearer to me that, without this general perspective, even the purest scientific research would suffer, inasmuch as, by alienating itself from the various and lively cultural developments going on, it would be condemned to the dryness which afflicts a plant shut up in a cellar without sunlight".[7]

Klein's document, *Gutachten*,[8] from May 1900 illustrates the evolution of his thoughts regarding education, and presents an overview of the cardinal points of his famous program for reforming mathematics teaching, which would be expressed publicly for the first time five years later at a congress held in Merano.[9]
The key points on which Klein based his reform movement are the following: he desired to bridge the gap between secondary and higher education and so he proposed

[5] See Giacardi 2006 and the texts of the programs on the website `http://www.mathesistorino.it/?page_id=564`.

[6] Regarding Klein and his movement to reform mathematics teaching, cf. Rowe, 1985; Schubring, 1989, 2008; Nastasi, 2000; Gario, 2006.

[7] F. Klein, *Göttinger Professoren. Lebensbilder von eigener Hand. 4. Felix Klein*, Mitteilungen des Universitätsbundes Göttingen, 5, 1923, p. 24.

[8] This document is transcribed in Schubring 1989, Appendix II.

[9] *Reformvorschläge für den mathematischen und naturwissenschaftlichen Unterricht*, Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht, 36, 1905, pp. 533-580.

transferring the teaching of differential and integral calculus, to the middle school level; he favoured a "genetic" teaching method, because developing a theory according to the way in which it is formed represents a good guide to scientific research; he believed that teachers should capture the interest and attention of their pupils by presenting the subject in an intuitive manner; he suggested highlighting the applications of mathematics to all the natural sciences; he believed in looking at the subject from a historical perspective; he argued that more space should be dedicated to the *Approximations-mathematik* that is, "the exact mathematics of approximate relations"; finally he firmly believed that the elementary mathematics from an advanced standpoint should play a key role in teacher training.

These tenets about the teaching of mathematics are intimately connected with Klein's vision of mathematics and mathematical research. In particular he believed that pure research had to be very closely tied to experimental research and he refused the axiomatic point of view, "the death of all sciences". He was convinced that the progress in science originates from the combined use of intuition and logic:

> "The science of mathematics may be compared to a tree thrusting its roots deeper and deeper into the earth and freely spreading out its shady branches to the air. Are we to consider the roots or the branches as its essential part? Botanists tell us that the question is badly framed, and that the life of the organism depends on the mutual action of its different parts".[10]

Moreover, Klein distinguished between *naïve intuition* and *refined intuition* and highlighted the fact that naïve intuition is important in the discovery phase of a theory (as an example he cites the genesis of differential and integral calculus), while refined intuition intervenes in the elaboration of data furnished by naïve intuition, and in the rigorous logical development of the theory itself (for example, in Euclid's *Elements*).[11]

Klein's epistemological vision of mathematics and his initiatives to improve mathematics teaching at the secondary and university levels in Germany arrived in Italy through various channels.[12]

First of all, towards the end of the nineteenth century a large number of young Italian mathematicians frequently attended German universities – in particular Leipzig and Göttingen, where Klein himself taught – in the course of postgraduate programs or on study trips: among these Giuseppe Veronese (1880–1881), Ernesto Pascal (1888–1889), Segre (summer 1891), Fano (1893–1894), and Enriques (1903) were the most noteworthy. They returned to Italy not only with new ideas about methods and areas of research, but also with new perspectives on teaching mathematics. Upon his return to Turin after several months of advanced study at Göttingen, Fano gave an enthusiastic account of Klein's work, referring also to Klein's teacher training seminars, observing

[10] F. Klein, *The Arithmetizing of Mathematics*, Isabel Maddison, trans. Bulletin of the American Mathematical Society 2, 8, 1896, pp. 241-249, at pp. 248-249.

[11] F. Klein, *On the mathematical character of space-intuition and the relation of pure mathematics to the applied sciences* (1893), pp. 41-50 in *Lectures on mathematics delivered from Aug. 28 to Sept.9, 1893 …at Northwestern University Evanston, Ill. by F. Klein, reported by A. Ziwet*, New York, Macmillan and C. 1894, at p. 42.

[12] See Giacardi 2010, pp. 2-4.

that, "We have a great deal to learn from Germany as far as the relationship between secondary and higher education is concerned".[13]
Klein himself visited Italy first in 1874 to meet the Italian mathematicians, to listen and to learn,[14] in 1878[15] and again in 1899, stopping over in Turin, Florence, Bologna, Rome, and Padua and meeting, among others, Segre, Enriques, Castelnuovo, Cremona, Veronese and Fano.[16]
Further, at the end of the 1800s several of Klein's writings touching upon mathematics education were translated into Italian, even before he became the president of the International Commission on Mathematical Instruction (Rome, 1908) and his ideas spread worldwide. Besides the *Erlanger Programm*, which Fano translated at Segre's request, the *Vorträge über Ausgewählte Fragen der Elementargeometrie*, was translated by Francesco Giudice at the request of the geometer Gino Loria, and the lecture "Über Arithmetisierung der Mathematik", appeared in the *Rendiconti del Circolo Matematico di Palermo*, translated by Salvatore Pincherle.[17]
Finally the rich correspondence between Klein and the Italian geometers, Segre, Fano, Loria, Enriques, and Castelnuovo,[18] bears witness to Klein's twofold influence in Italy, in scientific research and in mathematics teaching.
As Enriques would write twenty years later in his review of Klein's *Gesammelte mathematische Abhandlungen*, it was precisely the "tendency to consider the objects to be studied in the light of visual intuition" [19] that brought Klein and the Italian geometers so close together intellectually.

3. Corrado Segre and Teacher Training

Segre had been in epistolary correspondence with Klein since 1883, and considered him a *Maestro* at a distance. Klein's influence can be clearly perceived in his scientific work and in his methodological approach to mathematical research, but Klein

[13] G. Fano, *Sull'insegnamento della matematica nelle Universitá tedesche e in particolare nell'Universitá di Gottinga*, Rivista di matematica, 4, 1894, pp. 170-187, at p. 181.

[14] See for example, Klein to Cremona, 25 July 1874, 23 August 1874, 21 November 1874 in Menghini 1994, pp. 59-61.

[15] See, for example, Klein to Brioschi, 30 March 1878, Casorati to Brioschi, 4 November 1878, in *Francesco Brioschi e il suo tempo* (1824–1897) (2000), Milan, Franco Angeli, II Inventari, pp. 160 and 316.

[16] See Cremona to Fano, 21 March 1899 and Veronese to Fano, 21 March 1899, in *Fondo Fano*, Biblioteca matematica "G. Peano", Turin; Enriques to Castelnuovo, 17 March 1899 and 28 March 1899, in Bottazzini, Conte, & Gario, 1996, pp. 402 and 404; see also William Young, *Christian Felix Klein* -1949-1925, Proceedings of the London Royal Society, Series A 121(1928), pp. i-xix, at p. xiii.

[17] F. Klein, *Considerazioni comparative intorno a ricerche geometriche recenti*. Annali di matematica pura ed applicata, (2), 17, 1889, pp. 307–343; F. Klein, *Conferenze sopra alcune questioni di geometria elementare*. Torino, Rosenberg & Sellier 1896; F. Klein, *Sullo spirito aritmetico nella matematica*. Rendiconti del Circolo matematico di Palermo, 10, 1896, pp. 107–117.

[18] The letters are conserved in the dossier "*F. Klein*" of the Niedersächsische Staats-und Universitätsbibliothek of Göttingen and in the "Fondo Guido Castenuovo" of the Accademia Nazionale dei Lincei, Roma. See Luciano, Roero 2012 and P. Gario (ed.) *Lettere e Quaderni dell'Archivio di Guido Castelnuovo*: `http://archivi-matematici.lincei.it/Castelnuovo/Lezioni_E_Quaderni/menu.htm`.

[19] F. Enriques, (Review of) "F. Klein: *Gesammelte mathematische Abhandlungen, zweiter Band*." Periodico di matematiche, (4), 3, 1923, p. 55.

also left a strong imprint on Segre's conceptions of mathematics teaching.
This influence is evident in the lectures he gave at the *Scuola di Magistero* (Higher Teacher Training School) of Turin University; in fact, along with his courses of advanced geometry Segre also taught a course of mathematics for prospective teachers in this School for eighteen years, from 1887-88 to 1890-91 and from 1907-08 to 1920, the year when the Scuole di Magistero were suppressed. From that time until his death he taught the course in "complementary mathematics" that had just been established for the combined degree in mathematics and physics with the aim of preparing new graduates for teaching.
Leading Segre, a mathematician committed above all to preparing young people for research, to take an interest in the training of future teachers, was surely the influence of Klein, but other factors could have influenced him as well. First of all, the very close connection that Segre saw between research and teaching, which emerges quite clearly from his notebooks, led him to assign great importance to teaching methods. Further, the Turinese environment was very stimulating thanks to the presence of the Mathesis Association, a society of mathematics teachers, and the Peano School, which was particularly sensitive to problems related to teaching.[20] The example set by his mentor Enrico D'Ovidio, author of successful textbooks for secondary schools, and his friendship with Gino Loria, who quite soon became the referent abroad for questions regarding Italian education, may have influenced Segre as well.
Segre's teaching skills are attested to by forty handwritten notebooks (1888-1924) in which, every summer, he made a careful record of his lectures for the courses he was to teach the following autumn, dealing with different subjects each year. In fact, these not only constitute an important historical documentation of his research activity – of which, as Alessandro Terracini remarked, they are sometimes a preliminary stage, sometimes a reflection – but also provide extraordinary evidence of his gifts as a teacher.

The tenets which inspired Segre in advanced teaching are illustrated in his 1891 article *Su alcuni indirizzi nelle investigazioni geometriche. Osservazioni dirette ai miei studenti*.[21] He maintained that a good Maestro should invite his pupils only to deal with "relevant" problems and teach them to distinguish the significant questions from the sterile and useless ones; advise them to study, along with theories, their applications; urge them not to be "slave to one method", and not to restrict their scientific research within a too limited field, so as to be able to look at things "from a higher vantage point"; take due account of the didactic needs; suggest that they read the works of the great masters. These same issues are taken again into consideration in relation to secondary school teaching, in Segre's lessons at the Scuola di Magistero.

[20] See Luciano, Roero 2010.

[21] C. Segre, *Su alcuni indirizzi nelle investigazioni geometriche. Osservazioni dirette ai miei studenti*, Rivista di Matematica, 1, 1891, pp. 42-66 (Opere, 4, pp. 387-412). This article also reached American mathematicians thanks to the translation done by John Wesley Young and revised by Segre himself, who made several additions and modifications to it.

3.1. "Teach to discover"

Among Segre's handwritten notebooks, three in particular, *Lezioni di Geometria non euclidea* (1902-03), *Vedute superiori sulla geometria elementare* (1916-17) and [*Appunti relativi alle lezioni tenute per la Scuola di Magistero*], together with archival documents, show Segre's vision of mathematics and its teaching.[22]
Segre's lessons at the Scuola di Magistero were characterized by a threefold approach: theory, methodology, and practice. In fact, he took up anew the themes of elementary mathematics, making evident each time the connections to higher mathematics;[23] he also examined questions of methodology and didactics. Then, in the laboratories-classes, students were taught to impart real lessons, documented and stimulating.
In the notebook [*Appunti relativi alle lezioni tenute per la Scuola di Magistero*], Segre begins with some considerations on the nature of mathematics, the objectives of teaching, and the importance of intuition and rigor, then provides future teachers with some methodological instructions which are closely related to his particular way of approaching research, and are the fruit of his own teaching experience and of an attentive examination of legislative measures in various European countries and of educational issues debated at the time.

Between the two ways of addressing mathematics – considering it in relation to applications, or seeing it from an exclusively logical point of view – Segre's preferences tended towards the first. This first approach, typical of Klein, is characterised by three phases: gathering information derived from experience, putting the data obtained into mathematical form and proceeding to a purely mathematical treatment of the problem, and finally, translating the mathematical results into the form most suitable for the applications. With regards to the second approach, Segre cites Peano and Hilbert and observes:

> "Let us say immediately that this second line is of great importance, philosophically as well. It has made it quite clear what pure mathematics is; and has contributed to making various parts of mathematics more rigorous. But, by detaching itself from reality, there is a risk of ending up with constructions, which, even while logical, are too unnatural, and cannot be of lasting scientific importance" (pp. 13-14).[24]

For Segre, the aim of mathematics is to teach how "to reason well; not to be satisfied with empty words; to draw conclusions from the hypothesis, to reflect and discover on one's own; ...to speak precisely" (p. 42). In secondary teaching mathematics should not be considered an end in itself, "it must arise from the external world and then

[22] For further information and an overview of the international context see Giacardi 2003 and 2010.

[23] In the notebook *Vedute superiori sulla geometria elementare* (1916-17) Segre deals with the following subjects: non-Euclidean geometry, foundations of mathematics, elementary geometry and projective geometry, Analysis situs, geometrical constructions, linkages, problems that can be solved with straightedge and compass, algebraic equations that can be solved by extracting square roots, the problem of the division of the circumference, the problem of squaring the circle.

[24] This and successive page numbers refer to the notebook [*Appunti relativi alle lezioni tenute per la Scuola di Magistero*].

be applied to it" (p. 15); therefore the first approach to it must be experimental and intuitive, so that the student learns "not only to demonstrate truths already known, but to make discoveries as well, to solve the *problems* on his own" (p. 16), while "perfect rigor in certain things can be reached at a later time" (pp. 25-26).

Consequently, the objective of mathematics teaching is to develop not only the powers of reasoning but equally those of intuition; it is no coincidence that, as regards the method to be used, Segre prefers the *heuristic* method for presenting the subject, the *analytic* for the proofs, and the *genetic* for the development of theories. The first, the Socratic method, permits the student to discover mathematical truths on his own; the second allows him to enter into the mathematics "workshop" and understand the "why" of each step in a proof; the third represents a good guide to scientific research. Segre, however, never tired of underlining the importance of varying the methods, and above all, of choosing them according to "the subject, the pupils, and the time available" (p. 44).

The teacher should therefore seek the right balance between rigour and intuition. There are in Segre no specific philosophical reflections on the concept of intuition; for him, intuition is that used in scientific research: freedom of creative imagination, freedom of choice of methods, is "perceiving a truth spontaneously, without reasoning and without experiences, but it is the fruit of unconscious reasoning or experiences" (p. 15), based on previous knowledge that is unconsciously chosen and combined in new ways or that suggest analogies. How he understands the relationship between intuition and rigour in the practice of teaching emerges from the following considerations. According to Segre, the postulates on which the development of a theory is based must be intuitive and not necessarily independent. He also observes that in modern works are listed some "very obvious" postulates (for example, "the successor to a number is a number"), remarking that "a young person cannot understand the purpose of a series of such statements!" (p. 20). Further, he invites the teacher to use the postulates as need arises in a reasoning, without having to state them all at the beginning of the treatment of a theory. With regard to proofs, he observes that it is not necessary to prove propositions that are intuitively evident and that it can be useful to provide sketches of proofs rather than proofs that are rigorous but long and heavy:

> "A ... sketch, or non-rigorous proof may show how discoveries are made, how intuition works; or serve to provide an idea that is more synthetic, easier to remember, than the rigorous proof that will be explained later ...it is only necessary to warn the students that the proof presented is incomplete" (p. 25).

Concerning the definitions, Segre affirms:

> "to define for the young person the things that he already knows with a long discourse, is to bore him"(p. 46).
> "If we consider the exclusively logical point of view, the word line or curve, should be eliminated from elementary teaching because there are no means to define it. But that is absurd!" (pp. 18-19).

> "Don't give rigorous definitions, but clarifications, when the definition would be too difficult" (p. 46).

To highlight the difference between the intuitive approach and the logical approach in his lessons at the Scuola di Magistero, Segre takes as an example the textbook by Sebastiano Catania, *Trattato di Aritmetica ed Algebra* (1910) and compares the way the commutative property of the product of two numbers is presented there with the presentation used by Émile Borel in his textbook *Arithmétique* (1907) (p. 41). Catania, a follower of the Peano School, demonstrates it by induction,[25] while Borel illustrates it with an example that exploits the definition of the product as a sum.[26] On the other hand, Segre stresses the fact that it is important to underline the insufficiency of intuition for conceiving certain notions, as for example a curve that has no tangents (p. 43). As far as geometry is concerned, Segre agrees with Vailati's point of view, according to which teaching must be experimental and operative and must avail itself of teaching aids such as squared paper, drawing, and geometric models, that make it possible to "see certain properties that with deductive reasoning alone cannot be obtained".[27]

Moreover, like Klein, he believes that it is important to bridge the gap between secondary and university teaching by introducing, beginning in secondary schools, the concepts of function and transformation, and also to present some applications of mathematics to other sciences (physics, astronomy, political economy, geography, ...), in order to make the subject more interesting and stimulating.

In addition to considerations of a methodological nature, Segre does not hesitate to offer future teachers various bits of practical advice now and then, showing how aware he was of students' errors, bad habits, weak points, and idiosyncrasies:

> "Avoid being boring!" (p. 24);
> "Try to stimulate the activity of the student's mind" (pp. 26-27);
> "Sometimes satisfy the request for a proof which wouldn't be given, but which a more intelligent youngster can understand" (p. 27);
> "Vary the notations and figures. It shouldn't happen that a youngster does not know how to solve an equation only because the unknown is not called x. Or a geometric proof because the position of the figure has changed" (p. 28);
> "The calculations should not be too long, because there is no reason to try the patience of children" (p. 32);
> "Prepare the lesson perfectly ... Don't dictate: use a textbook ... Be patient with the students; repeat if they have not understood; don't be aghast at errors; try to persuade the students ... that they needn't have a gift for mathematics" (p. 42).

Segre's notebook related to the courses for the Scuola di Magistero, includes an annotated bibliography divided into sections, with references to texts then very recent

[25] S. Catania, *Trattato di Aritmetica ed Algebra ad uso degli Istituti Tecnici*, Catania, N. Giannotta, 1910, p. 32.

[26] E. Borel, *Arithmétique*, Paris, A. Colin, 1907, pp. 28-29.

[27] C. Segre, *Su alcuni indirizzi nelle investigazioni geometriche*, cit. p. 54.

both Italian and foreign. In addition to specific books on various topics of elementary mathematics, there are texts on didactics, as Segre himself calls them, textbooks for secondary schools and workbooks of exercises, books on the history of mathematics and on recreational mathematics. In addition to Klein, Segre's principal points of reference were C. A. Laisant, E. Borel, J. Hadamard and H. Poincaré in France, and P. Treutlein and M. Simon, in Germany, mathematicians all critical of a mathematics teaching too marked by logical rigour.

Segre's contribution to the field of mathematics education remained limited to the lessons at the Scuola di Magistero, nevertheless for eighteen years he trained the mathematics teachers who came out of the University of Turin, contributed to the spread of Klein's vision of mathematics teaching among his students, and by his very example transmitted a certain way of teaching mathematics, both at the university and the secondary level, exploiting intuition, encouraging creativity, using more than one method, and establishing connections between different sectors in a unitary vision of mathematics. Moreover, through individual discussions with his students as well as his university lectures, he had communicated his vision of mathematics teaching to his School, although each of its members interpreted it in their own way, according to their different individual experiences.

4. Guido Castelnuovo: the involvement in education as a social duty

Thanks to the intervention of Segre, Castelnuovo was called to Turin as assistant to D'Ovidio in 1887, and remained until 1891, when he obtained a professorship in Rome. As is well known, the scientific collaboration between the two young mathematicians – at the time, Segre was 28, Castelnuovo 26 – led to the creation of the Italian line of research on the geometry of algebraic curves and laid the bases for all of Italian algebraic geometry.[28] After leaving Turin, Castelnuovo kept up the correspondence with his friend and mentor that had begun in 1885, and in Rome inaugurated the extraordinary scientific fellowship with Enriques.

Already at this time in his correspondence, along with questions regarding research there appear topics concerning education,[29] but it is in 1907 that Castelnuovo began to turn his thoughts and efforts to the improvement of mathematics teaching in secondary schools, becoming involved also at an institutional level. That year, in fact, appeared the article entitled "Il valore didattico della matematica e della fisica", which can be considered a manifesto of Castelnuovo's thinking on education. That same year Castelnuovo was involved in the organisation of the International Congress of Mathematicians, which was held in Rome from April 6 to 11, 1908, and paid a great deal of attention also to the section devoted to education, with the help of Vailati, who at the time was working with the Royal Commission on the project to reform secondary

[28] Brigaglia, Ciliberto 1995, §1.5.

[29] The 255 letters from Segre to Castelnuovo dating from 1885 to 1905 can be read in P. Gario (ed.) *Lettere e Quaderni dell'Archivio di Guido Castelnuovo*: `http://operedigitali.lincei.it/Castelnuovo/Lettere_E_Quaderni/menu.htm`
(see also Bottazzini, Conte, Gario 1996, pp. 669-678). For the letters from Enriques to Castelnuovo, see Bottazzini, Conte, Gario 1996.

schools.[30] During the congress an international committee dedicated to issues pertaining to mathematics teaching was created, the International Commission on Mathematical Instruction (later ICMI):[31] its first president was Klein, and Castelnuovo, Enriques and Vailati were the Italian delegates. It is thus natural that the contacts with Klein, who he had already met in 1899, became more intense.
In 1909, during the Congress of the Mathesis Association in Padua Castelnuovo explicitly proposed following Klein's example with regard to teacher training:

> "At Klein's suggestion, during the spring holidays a number of German universities hold short courses for Middle school teachers. Couldn't we too set up similar courses in our universities?".[32]

In the years immediately following he himself began to include in his courses in higher geometry in Rome a number of topics designed specifically for the cultural training of future mathematics teachers following the examples of Klein and Segre. Of particular interest from this point of view are the following notebooks: *Geometria non-euclidea* (1910–11), *Matematica di precisione e matematica di approssimazione* (1913–14), *Indirizzi geometrici* (1915–16), *Equazioni algebriche* (1918–19) and *Geometria non-euclidea* (1919–20).[33] In the introduction to the 1913–1914 course on the relationship between precise and approximate mathematics, Castelnuovo explicitly discusses the various ways in which future teachers can be trained and quotes Klein:

> "The educational value of mathematics would be much enriched if, in addition to the logical procedures needed to deduce theorems from postulates, teachers included brief digressions on how these postulates derive from experimental observations and indicated the coefficients with which theoretical results are verified in real experience ...The relationship between problems pertaining to pure mathematics and those pertaining to applied mathematics is very interesting and instructive. Klein, who dedicated a series of lectures to the subject (1901), describes the first of these as problems of 'precise mathematics' and the second as problems of 'approximate mathematics'. In this course we will ...more or less follow the general outline of Klein's course.[34] Klein also had another reason for pursuing this line of enquiry, that is, his desire to bridge the gap between mathematicians engaged in pure research and those who have to solve problems relating to applied mathematics."[35]

[30] See G. Castelnuovo to G. Vailati, s. l., February 16, 1907, and D. E. Smith to G. Loria, New York, January 12, 1906, *Fondo Vailati*, Library of Philosophy, University of Milan.

[31] See Giacardi, 2008: `http://www.icmihistory.unito.it/19081910.php`.

[32] G. Castelnuovo, *Sui lavori della Commissione Internazionale pel Congresso di Cambridge. Relazione del prof. G. Castelnuovo della R. Università di Roma*, in *Atti del II Congresso della Mathesis - Società italiana di matematica*, Padova, 20-23 Settembre 1909, Padova, Premiata Società Cooperativa Tip. 1909, Allegato F, pp. 1–4, at p. 4.

[33] The notebooks can be read in P. Gario (ed.) *Lettere e Quaderni dell'Archivio di Guido Castelnuovo*, cit. See also Gario 2006.

[34] Castelnuovo is referring to F. Klein, *Anwendung der Differential-und Integralrechnung auf Geometrie*, Leipzig Teubner 1902.

[35] G. Castelnuovo, *Matematica di precisione e matematica di approssimazione*, 1913–14, pp. 2–3.

Thus, there is no doubt that Klein's influence was important, but Castelnuovo's interest in educational issues also arose from social concerns, as he himself affirmed in the lecture he gave in Paris in the stead of ICMI President Klein and at his express request:

> "Nous nous demandons parfois si le temps que nous consacrons aux questions d'enseignement n'aurait pas été mieux employé dans la recherche scientifique. Eh bien, nous répondons que s'est un devoir social qui nous force à traiter ces problèmes."[36]

4.1. "Break down the wall separating schools from the real world"

Castelnuovo's approach to education grew out of a lucid critique of the Italian school system: in his opinion the teaching of mathematics was too abstract and theoretical, all reference to practical application was neglected and an excessive specialization of different areas led to a distorted cultural perspective.
In order to define exactly what secondary schools should be offering to young people, Castelnuovo asked himself the following three questions: At whom is middle school education aimed?; What should the ultimate goal of schooling be?; What skills should teaching develop? He believed that schools should cater above all to young people aiming to go into one of the so-called "free" professions "both because they constitute the majority of school pupils and because the progressive development of the country will rest mainly on their shoulders".[37] The primary aim of middle schools should be to form the future member of civil society, because "education cannot be truly effective if it is not aimed at average levels of intelligence and if it is unable to create that refined democracy which forms the basis of every modern nation".[38] The qualities which teachers must foster and cultivate in their pupils are the creative imagination, the spirit of observation and the faculties of logic. Excessive rigour is to be avoided:

> "Middle schools should not furnish [their pupils] with knowledge so much as with a desire and a need for knowledge; they should not seek to provide an encyclopaedic knowledge, but must only offer a clear, although necessarily very limited, idea of the principal questions of the various branches of knowledge, and of some of the methods which have been employed in tackling them. [...] Of course, this kind of teaching will not be sufficient to provide middle school students with preparation specific to one or another of the faculties of the university. However, this is not the aim of middle schools. They serve simply to provide students with the aptitude to move on to more advanced studies."[39]

[36] G. Castelnuovo, *Discours de M.G. Castelnuovo*, L'Enseignement Mathématiques, 16, 3, 1914, pp. 188–191, at p. 191.

[37] G. Castelnuovo, *La scuola nei suoi rapporti colla vita e colla Scienza moderna*, in *Atti del III Congresso della Mathesis - Società italiana di matematica, Genova, 21-24 ottobre 1912*, Roma, Tip. Manuzio, 1913, pp. 15–21, at pp. 18–19.

[38] Castelnuovo, *Sui lavori della Commissione Internazionale pel Congresso di Cambridge*, cit. p. 4.

[39] G. Castelnuovo (1910), *La scuola media e le attitudini che essa deve svegliare nei giovani*, in *Guido Castelnuovo, Opere Matematiche. Memorie e Note*, vol. III, Roma, Accademia dei Lincei, 2004, pp. 21–30.

In the article "Il valore didattico della matematica e della fisica", mentioned above, the placing of mathematics and physics side by side is by no means coincidental.[40] Here, in fact, Castelnuovo emphasises the importance of observations and experiment, the usefulness of constantly confronting abstraction with reality because "it in fact makes it possible to clarify the two different meanings given to the adjective exact in the theory and in the practice", and the importance of practical application as a means of "shedding light on the value of science". Furthermore, he claims that heuristic procedures should be favoured for two reasons: "the first, and the most important reason, is that this type of reasoning is the best way to attain to truth, not just in experimental sciences, but also in mathematics itself"; the second is that it is "the only kind of logical procedure that is applicable in everyday life and in all the knowledge involved with it."[41]
He concludes his article by recommending that teachers draw on the history of science so that young people understand the relative and provisional nature of every theory.

To illustrate more incisively his conception of mathematics teaching, Castelnuovo often introduced veritable slogans in his speeches and in his articles: "Rehabilitate the senses"; "Break down the wall separating schools from the real world"; "Teaching should proceed hand in hand with nature and with life".

4.2. Various directions of Castelnuovo's commitment to education

Castelnuovo's commitment to education manifested itself in various forms: in his activities in the ICMI and the Mathesis Association, of which he was president from 1911 to 1914, in the courses he taught at university, in the various articles he dedicated to issues relating to mathematics teaching, and in the syllabi of *liceo moderno* he designed for secondary education.

As a delegate and, later, first as a member of the Central Committee of the ICMI and then as vice-president, Guido Castelnuovo built up an international network and promoted the exchange of information about the new movements for reform in Europe, in particular that proposed by Klein, whose methodological approach he wholeheartedly endorsed.[42] Further, he guided the work of the Italian subcommission of the ICMI, encouraging its members not to "occupy themselves only with statistical data" but to "turn the investigation to more elevated fields and to treat pedagogical and psychological questions." [43]

As president of the Mathesis Association, he inserted into the *Bollettino* of the Mathesis summaries of the activities of the subcommission, translations of lectures, inquiries into problems concerning mathematics teaching in the various orders of schools. He also encouraged debates regarding method. For example he wrote to Giovanni Vacca:

---

[40] See Brigaglia 2006.

[41] This and the above quotations are from G. Castelnuovo, *Il valore didattico della matematica e della fisica*, Rivista di Scienza, 1, 1907, pp. 329–337, respectively at p. 336 and at p. 333.

[42] See Giacardi 2008.

[43] Castelnuovo, *Sui lavori della Commissione Internazionale pel Congresso di Cambridge*, cit. p. 2.

> "Almost unexpectedly and against my will, I have been elected president of the Mathesis. I accept the nomination only because I think that it might be helpful for the affairs of the Italian Commission for mathematics teaching, for which the Bollettino of the Math[esis] will become the publishing organ. I would like keep the level of the Bollettino high, reducing to a minimum the Byzantine discussions in which secondary teachers too often delight. I am therefore very much counting on your cooperation."[44]

When in 1911 Minister of Education Luigi Credaro set up a *liceo moderno*, in which Greek was replaced with a modern language (German or English) and more attention was dedicated to the scientific subjects, Castelnuovo drew up the mathematics syllabus and related instructions.[45] He put a number of Klein's proposals into practice by introducing the notion of function and the concepts of derivative and integral, attaching a greater importance to numerical approximations, and coordinating mathematics and physics teaching. He wrote:

> "But if we truly wish the middle school student to feel an inspiring breeze in this modern mathematics, and perceive something of the grandeur of its whole structure, it is necessary to speak to him of the concept of function and show him, even summarily, the two operations that constitute the foundation of infinitesimal calculus. In this way, if he will have a scientific spirit, he will acquire a more correct and balanced idea of the exact sciences nowadays ...If the pupil's mind is more disposed towards other subjects, he will at least find mathematics to be, instead of a logical drudge, a set of methods and results which can be easily applied to concrete problems." [46]

On that occasion Castelnuovo wrote to Klein:

> "A propos de l'enseignement, certain que vous agréerez la nouvelle, je vais vous communiquer que les programmes (modernes) de l'enseignement mathématique que j'ai fait adopter dans les lycées modernes, ont été si bien accueillis que le Ministére de L'Instr. P. pense maintenant de les introduire même dans les lycées classiques et dans les instituts techniques, en développant davantage, dans ces dernières écoles, le programme de calcul infinitésimal." [47]

He also presented the programs to the international community during the ICMI meeting held in Paris in 1914.[48] Unfortunately, the *liceo moderno* was short-lived. The reorganisation of secondary schools was introduced in 1923 by the Gentile Reform in

[44] See the letter of G. Castelnuovo to G. Vacca, Roma, January 27,1911, in Nastasi, Scimone 1995, p. 46.

[45] See *Ginnasio - Liceo Moderno. Orario - Istruzioni - Programmi*, 1913, in `http://www.mathesistorino.it/wordpress/wp-content/uploads/2012/09/liceomod.pdf`.

[46] G. Castelnuovo, *La riforma dell'insegnamento matematico secondario nei riguardi dell'Italia*, Bollettino della Mathesis, XI, 1919, pp. 1-5, at p. 5.

[47] See the letter of G. Castelnuovo to F. Klein, Rome, March 10, 1915, in Luciano, Roero 2012, p. 212.

[48] G. Castelnuovo, *Italie*, L'Enseignement Mathématique, 16, 1914, p. 295. The programs for the liceo

completely different terms: the liberal democratic culture was defeated by new political trends – firmly opposed by Castelnuovo – and by the triumph of Neo-Idealism. Castelnuovo would reformulate some of his cherished ideas in *Progetto di riforma dell'insegnamento secondario* that he presented in 1947 for the political party called *Partito d'Azione*. Here he maintained the importance of education's being developed in a liberal climate, advancing the proposal of a single, common middle school without Latin, a proposal dictated, this time as well by social needs: "The social needs are the most pressing; we must first of all satisfy these, as far as possible, and formulate the cultural problem accordingly."[49]

In Castelnuovo, as in Segre, we find no lofty philosophical or epistemological reflections on the foundations or on problems of rigour and intuition; these appear not to interest him.[50] What he did take to heart was a mathematics teaching, and in particular a geometry teaching, that was strictly tied to reality, in which the deductive aspect was only one of three stages to be gone through: a first stage consists in "the passage from reality to the symbolic scheme", with resort to experience and intuition; in the second stage work must be done on the symbols by means of logical procedures to deduce new truths; and in the third the abstract propositions are translated into results that can be applied to reality.[51] For this reason, in his opinion, a good teacher had to develop above all "the creative imagination", which grows out of the union of intuition and the spirit of observation. In the same way, a good textbook had to find the balance between intuition and rigour, as he forcefully maintained, for example, in the debate with Catania, already cited by Segre, regarding algebra textbooks for secondary schools:

> "If I had to teach in middle schools, I would avoid adopting a textbook in which, along with procedures that are perfectly rigorous, ample space was not given to intuition and experience. Not because I refuse to recognise the immense value of deductive logic ... but I would be blind if I did not see that with logic alone science would never have been born!". [52]

Again, in the comparative investigation that he undertook for the ICMI in 1911 regarding rigour in the teaching of geometry in classical schools, he was clearly inclined towards a deductive development of geometry teaching which, however, began from empirical bases.[53]

There are two other significant aspects in Castelnuovo's work that must be made

moderno and the instructions regarding methodology were also translated into French: see G. Loria, *Les Gymnases-lycées "modernes" en Italie*, Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht aller Schulgattungen, 1914, pp. 188–193.

[49]G. Castelnuovo, *Progetto di riforma dell'insegnamento secondario*, in Castelnuovo (1944), *Opere Matematiche* 2004, cit. p. 410.

[50]See, for example, the letters from F. Enriques to G. Castelnuovo of May 1896, particularly that of 4 May 1896, in Bottazzini, Conte, Gario 1996, pp. 260–261.

[51]Castelnuovo, *Il valore didattico della matematica e della fisica*, cit., pp. 331–332.

[52]G. Castelnuovo, *Risposta ad un'osservazione del Prof. Catania*, Bollettino della Mathesis, 1913, pp. 119–120.

[53]G. Castelnuovo, *Commissione internazionale dell'insegnamento matematico – Congresso di Milano*, Bollettino della Mathesis, 1911, pp. 172–184, and L'Enseignement Mathématique 13, 1911, pp. 461–468.

evident: the importance attached to the formation of a "cultured democracy", where the sciences were considered as important as letters; and the need for an adequate scientific and didactic training for secondary school teachers,[54] for whom he, among other things, made room in the activities of the Italian subcommission of the ICMI.

A final point that must be underlined is his firm opposition to the Gentile Reform, which led to his refusal to collaborate with the minister on drafting of the programs for secondary schools,[55] and which was clearly explained in his report "Sopra i problemi dell'insegnamento superiore e medio a proposito delle attuali riforme",[56] compiled for a commission of the Accademia dei Lincei, of which Vito Volterra was president.

5. Federigo Enriques: Teacher training and scientific humanitas[57]

As is well known, after earning his degree Enriques' aspiration was to specialise with Segre in Turin, but instead he was appointed to Rome where he began his extraordinary fellowship with Castelnuovo. Their collaboration led to the publication of important works on algebraic surfaces[58] and developed into a lasting friendship.

However, Enriques did come to Turin, first in November 1892 for several weeks and then from November 1893 to January 1894.[59] The stay in Turin was intense from a scientific point of view, but also inspired his first reflections on the foundations of geometry, which he would later develop during the time spent in Bologna, where he was called to teach projective and descriptive geometry. In January 1922 he transferred to Rome to teach complementary mathematics and then higher geometry, thanks to Castelnuovo having renounced that professorship.

In addition to strictly scientific interests, the two friends also shared a profound attentiveness to education, although they were motivated by different reasons. While Castelnuovo's engagement in educational issues sprang from social concerns, Enriques' involvement was rooted in his interests in philosophical, historical, and interdisciplinary issues and his studies on the foundations of geometry. In 1896, the teaching of projective geometry at the University of Bologna stimulated him to study the genesis of the postulates of geometry, taking the psychological and physiological

[54] Regarding this point, see, for example, *Atti del II Congresso della Mathesis - Società italiana di matematica. Padova 20.23 Settembre 1909*, Padova, Premiata Società Cooperativa Tipografica, 1909, pp. 42–43, and Castelnuovo's *Relazione*, appended to the minutes of the meeting of the Facoltà di Scienze Matematiche, Fisiche e Naturali of the University di Rome of 14 March 1922, in Gario 2004, pp. 117–119.

[55] Assisting the minister in the preparation of the mathematics program was Gaetano Scorza, an active member of the Mathesis Association and one of the Italian representatives in the International Commission on Mathematical Instruction; see *Relazione del Congresso di Livorno, 25-27 Settembre 1923*, Periodico di matematiche, s. IV, 3, 1923, p. 465.

[56] See G. Castelnuovo, *Sopra i problemi dell'insegnamento superiore e medio a proposito della attuali riforme*, in Castelnuovo, *Opere Matematiche* 2004, cit. pp. 358–367.

[57] For further details about this, see Giacardi 2012.

[58] Brigaglia, Ciliberto 1995, Chap. 1, 4; Bottazzini, Conte, Gario 1996.

[59] See, for example, the letters of F. Enriques to G. Castelnuovo, November, 6, 9, 1892 and November, 23, 1893, in Bottazzini, Conte, Gario 1996, pp. 3-4 and p. 44.

studies of H. Helmholtz, E. Hering, E. Mach, W. Wundt, as his starting point.[60]

From the correspondence, writings and documents conserved in the archives of the University of Bologna it is possible to see how already in these early years Enriques had turned his attention to the needs of education, stressing in particular the refusal to resort to artifices in the proofs; the importance of using intuition; the connections between elementary and higher mathematics; the use of the history of mathematics as a tool for understanding the genesis of the concepts presented, and a unified vision of science and culture.[61]

In the evolution of Enriques' cultural project and his vision of mathematics teaching, along with his experience in teaching, an important role was played by the influence of Klein.[62] During Klein's second trip to Italy in 1899 the principal theme of conversation was the psychological genesis of postulates[63] and when Klein invited him to write a chapter on the foundations of geometry for the *Encyklopädie der mathematischen Wissenschaften*, Enriques reached him in Göttingen in 1903 to discuss this subject:

> "In addition to talking about the foundations of geometry, we discussed didactic issues at length, and in just a few hours I learned a great deal from him about a lot of things I knew nothing about – specifically about the way in which mathematics teaching is developing in England and Germany".[64]

It was thanks to Klein's that a German translation of Enriques' *Lezioni di geometria proiettiva* was published in 1903. In his introduction to this book, Klein expresses particular appreciation for Enriques' treatment of the subject, which "is always intuitive, but thoroughly rigorous", and underlines the impact of this kind of research on didactics, writing

> "Italian researchers are also well ahead of us from a practical point of view. They have by no means disdained exploring the didactic consequences of their investigations. The high quality textbooks for secondary schools which came out from this exploration could be made available to a broader audience through good translations. And it would seem particularly desirable in Germany when we consider that our own textbooks are completely out of touch with active research".[65]

---

[60] See F. Enriques, *Lezioni di Geometria proiettiva*. Bologna, Zanichelli 1898; *Sulla spiegazione psicologica dei postulati della geometria*, Rivista di Filosofia 4, 1901, pp. 171–195 (rpt. Federigo Enriques, *Memorie scelte di Geometria*, Roma, Accademia Nazionale dei Lincei, vol. II, 1959, pp. 145–161); *Problemi della Scienza*, Bologna, Zanichelli 1906; and F. Enriques to G. Castelnuovo, 4 May 1896, in Bottazzini, Conte, Gario 1996, pp. 260–261.

[61] See, for example, Bottazzini, Conte, Gario 1996, p. 224; Chisini 1947, p. 119; *Registri e Relazioni*, in Archivio Storico dell'Università di Bologna.

[62] See Nurzia 1979; Israel 1984; Giacardi 2012, pp. 223-229 and Appendix 2, "Letters to Klein".

[63] F. Enriques to G. Castelnuovo, 28 March 1899, in Bottazzini, Conte, Gario 1996, p. 404.

[64] F. Enriques to G. Castelnuovo, 24 October 1903, in Bottazzini, Conte, Gario 1996, p. 536.

[65] F. Klein, *Zur Einführung*, in F. Enriques, *Vorlesungen über projektive Geometrie*, Introduction (*Zur Einfürung*) by F. Klein. H. Fleisher, trans. Leipzig, Teubner, 1903 (2nd. ed. 1915), p. III.

5.1. Enriques' cultural programme and epistemological assumptions at the basis of his idea of Mathematics Teaching

From the first years of the twentieth century Enriques had in mind a very precise cultural program, one in which active research in the field of algebraic geometry and philosophical, psychological and historical reflections are all closely intertwined. Enriques' aim was to communicate to his intended audience – mathematicians, scientists, philosophers, and educators – his vision of a scientific *humanitas* in which the boundaries between disciplines were overcome and the abyss between science and philosophy was bridged. The history of science constituted the way for achieving this end, or at least it was the tool used by Enriques in his university courses from the very first years, and over time gradually became a very important one in the various initiatives aimed at teacher training.

It is not easy to outline in a few strokes the epistemological vision on which all of Enriques' scientific work was founded, so I will confine myself to indicating the most important factors which inspired his idea of mathematics education.

– Enriques held a dynamic and genetic view of the scientific process, describing it as a "process of continuous development, which establishes a generative relationship between theories and perceives in their succession only an approximation to truth."[66] In such a vision of science, errors become valuable as well, because in the dynamic process of science truth and error are constantly mixed: "every error always contains a partial truth that must be kept, just as every truth contains a partial error to be corrected".[67]

  As a consequence of this idea, Enriques criticises the tendency to present a mathematical theory in strictly deductive manner, because in this way though it appears closed and perfect, it leaves no room for further discovery. Instead, teachers should approach problems with a number of different methods, paying attention to the errors which have allowed science to move forward and indicating those questions which remain open as well as new fields of discovery.[68]

– These views on science are connected to Enriques' conception of the nature of mathematical research – typical of the Italian school of algebraic geometry – as something aiming above all at discovery and particularly emphasizing the inductive aspects and intuition: 'As a rule, the main thing is to discover ... A posteriori it will always be possible to give a proof' ..., [which] 'translating the intuition of the discoverer into logical terms, will provide everyone with the means to recognise and verify the truth'.[69]
  Much has been written on the working method of the Italian geometers, and

[66] F. Enriques, *Scienza e razionalismo*, Bologna, Zanichelli, 1912, p. 132.

[67] F. Enriques 1911, *Esiste un sistema filosofico di Benedetto Croce?*, Rassegna contemporanea, 4.6, 1911, pp. 405–418, at p. 417.

[68] See F. Enriques [Adriano Giovannini], *L'errore nelle matematiche*, Periodico di Matematiche 4, 22, 1942, pp. 57–65.

[69] F. Enriques, O. Chisini, *Lezioni sulla teoria geometrica delle equazioni e delle funzioni algebriche*, 4 vols. Bologna, Zanichelli, 1915–1934, II 1918, pp. 307, 318.

about Enriques in particular, so here I will limit myself to underlining by means of a quotation the importance that he attached to intuition in scientific research:

> "The faculty which comes into play in the construction of science and which thus expresses the actual power of the mathematical spirit is intuition. ...There are in any case different forms of intuition. The first is the intuition or imagination of what can be seen. ...But there is another form of intuition that is more abstract, that – for example – which makes it possible for the geometer to see into higher dimensional space with the eyes of the mind. And there is also a sense of formal analogies which, in the work of many analysts, takes the place of the visual representation of things. ...[I]ntuition protracts and surpasses itself in the unifying power of reason, which is not something exclusive to the mathematician, but – in every field of science and application – marks the greatest reaches of the spirit".[70]

This belief is naturally reflected in the style of teaching, which should, according to Enriques, take into account the inductive as well as the rational aspect of theories. Logic and intuition represent two inextricable aspects of the same process, therefore teachers should find the right balance between the two: the important thing is to distinguish clearly between empirical observation and intuition on the one hand, and logic on the other. Teaching should above all take into account "large scale logic" (which considers the organic connections in science), not so much the "small scale logic" (refined, almost microscopic analysis of exact thought), and prepare young people gradually to develop a more refined and rigorous analysis of thought:

> "It is of no use to develop with impeccable deduction the series of theorems of Euclidean geometry, if the teacher does not go back to contemplate the edifice constructed, inviting the students to distinguish the truly significant geometric properties from those which are valuable only as links in the chain".[71]

With regard to the fact that many Italian teachers resisted the introduction of methods that were more intuitive and empirical, lamenting that a certain incompleteness and a non-rigorous way of reasoning is inherent in these, Enriques observed with a touch of humour:

> "Resisting the ideas that ...relate to the eye, the ear, the sense of touch, and seeing in sensations, not the doors to knowledge, but only occasions for sinful errors, this strange chastity of mathematical logicians brings to mind Plotinus and those Christian ascetics of the Middle Ages who were ashamed of having a body".[72]

[70] F. Enriques, *Le matematiche nella storia e nella cultura*. Bologna, Zanichelli, 1938, pp. 173–174.
[71] F. Enriques, *Insegnamento dinamico*, Periodico di Matematiche 4, 1, 1921, pp. 6–16, at p. 10.
[72] Enriques, *Le matematiche nella storia e nella cultura*, cit, p. 145.

– Moreover, for Enriques science is a "conquest and activity of the spirit, which ...merges in the unity of the spirit with the ideas, feelings and aspirations which find expression across all the different aspects of culture"[73], so it is important to establish links between scientific knowledge (mathematics, physics, biology, etc.) and other intellectual activities (psychology, physiology, philosophy, history, etc.). In this Enriques ran counter to Croce and Gentile, the leading exponents of Italian Neo-Idealism, who tended to devalue science, recognising in it only a practical function and a role that was completely instrumental, and separating it from the world of philosophy and culture.[74] For Enriques, the fact that science does not have purely utilitarian goals does not imply a separation between pure and applied science, but means that scientific research is valuable in itself, and does not necessarily have to aim at applications. Like Klein, he believed it was useful and necessary to maintain close ties between abstract science and applied sciences because pure sciences offer instruments that are needed for the purposes of applied science, and in their turn, applied sciences perform functions that are essential for stimulating the development of theoretical sciences, as history makes amply clear.[75]

  As a consequence of this conviction, according to Enriques, the duty of the teacher consists in inducing the students to acquire knowledge for themselves, because only in this way can they arrive at the true comprehension of mathematics. So he suggests using the Socratic method, which consists in conversing with them, acting 'a little ignorant' and, through dialogue and a guided search, leading them to a personal discovery of mathematical truth. Furthermore teachers should transmit a unified vision of knowledge to their pupils, because only by overcoming narrow specialisation can mathematics achieve its true humanistic and formative value.

– Another central aspect of Enriques' epistemological vision is his belief that scientific developments can only be fully understood in their historical connection:

> "A dynamic vision of science leads us naturally into the territory of history. The rigid distinction that is usually made between science and history of science is founded on the concept of this [history] as pure literary erudition; ...But the historical comprehension of scientific knowledge that aims at ...clarifying the progress of an idea has a very different meaning. ...Such a history becomes an integral part of science."[76]

  Furthermore, history also offers the cultural legitimisation of the function of mathematics, and thus for Enriques has a central educational role in both teacher

[73] F. Enriques, *L'importanza della storia del pensiero scientifico nella cultura nazionale*, Scientia 63, 1938, pp. 125–134, at p. 130.

[74] See for example, Israel 1984; Pompeo Faracovi 2006.

[75] F. Enriques, *Il significato umanistico della scienza nella cultura nazionale*, Periodico di Matematiche 4, 4, 1924, pp. 1-6, at p. 4.

[76] Enriques, Chisini, *Lezioni sulla teoria geometrica delle equazioni*, cit., I, p. XI.

training as well as in teaching proper: future teachers should study the origins of each theory, together with its developments, not some static formulation and young people too should be "educated in the masterpieces of the masters".[77] The history of science can also constitute an important auxiliary tool for education in making it possible to better understand certain concepts or properties.

5.2. The Battle for a Scientific Humanitas in the first decades of the twentieth century

Enriques used various strategies for carrying out his cultural project and imposing his vision of a scientific *humanitas*. These followed different lines depending on context (world of culture, institutions, editorial projects, university courses) and the intended audience (university professors, secondary school teachers, philosophers and scientists in general). I will limit myself to mentioning the initiatives with the strongest links to teaching.[78]

– In 1900 Enriques published the *Questioni riguardanti la geometria elementare*, specifically designed for teacher training purposes and inspired by Klein's *Vorträge über Ausgewählte Fragen der Elementargeometrie* (1895). The topics treated were congruence, equivalence, the parallel theory and non Euclidean geometry, problems that could or could not be solved with straightedge and compass, and the constructibility of regular polygons. In fact, although for Enriques Euclidean geometry remained "the most effective tool for educating the mind, the most consistent with geometric reality", he, like Klein, nevertheless believed that the teaching of geometry could "take advantage of the progress made, in the field of the elements as well, by a more mature criticism and recent developments in higher mathematics", and that "the teacher entrusted with secondary school education must possess a much broader knowledge of such progress so that his work is inspired by much larger perspective".[79] This work also shows that Enriques did not at all disdain "the microscope" (small scale logic), and that he was aware of its usefulness in science, but while he held that questions about the foundations were important for teacher training, he was convinced that it was necessary to maintain a knowing balance between rigour and intuition in secondary teaching.[80]

– In 1903 Enriques published, with Ugo Amaldi, the *Elementi di geometria*, the first in a long and successful series of textbooks. The treatment followed a "rational inductive" method: beginning with a series of observations, the authors enunciate certain postulates from which the theorems that depend on them are developed by logical reasoning; from these theorems, they then continually return to observations or intuitive explanations. In this case as well Enriques ac-

[77] Enriques, *Insegnamento dinamico*, cit. p. 16.
[78] For further details see Giacardi 2012.
[79] F. Enriques, *Questioni riguardanti la geometria elementare*. Bologna, Zanichelli, 1900, pp. I–II.
[80] Enriques, *Insegnamento dinamico*, cit., p. 11.

knowledged Klein's influence,[81] and Klein mentions this textbook in his essay on geometry teaching in Italy, *Der Unterricht in Italien*, praising the authors for having taken didactic requirements into consideration, thus reconciling logical rigour and intuition.[82]

– In 1906 Enriques founded the Italian Philosophical Society and was its president until 1913.

– In 1906 on the occasion of the congress of the Federazione Nazionale Insegnanti Scuola Media held in Bologna, he explained his opinion regarding teacher training. He then suggested the establishment of a *pedagogical degree* in addition to the *scientific degree*: the first two years of study would be dedicated to acquiring basic knowledge of the discipline, and by the end of that time, a distinction would be made between those who intended to dedicate themselves to research and those who wanted to teach. For the future teachers, the next two years would be aimed at providing professional training by means of "1) courses on those parts of science that aim at a more profound understanding of the elements, 2) lectures on concrete questions of pedagogy that interest the various areas of teaching, particularly in relation to the analysis of the textbooks, 3) exercises comprising practice teaching, partly in the university and partly in secondary schools, drawing, and experimental technique".[83]

– In 1906, during the first International Congress of Philosophy in Milan, Enriques proposed a project to reform the Italian universities, and in accordance with his strategy, that same year explained his point of view to the middle school teachers in Bologna, to the mathematicians and scientists in his 1908 article in the *Rivista di Scienza*, and finally to university professors in 1911. His project had grown out of the ascertainment of the defects of the Italian university system which had, in his opinion, serious repercussions for research, teaching and the work world: the lack of interaction between the various faculties; the excessive fragmentation of disciplines with programs too heavy; the tendency of each professor to defend his own discipline favouring the pre-eminence of already consolidated areas of research over those which were interdisciplinary or unexplored. The solution he proposed was that of conjoining in a single faculty of philosophy all of the theoretical disciplines: mathematics, physics, physiology, history, law, economy, etc., so as to "correspond to the synthesis required by renewed philosophical consciousness and practical life, as opposed to the scientific-educational particularism of the previous era".[84] He also proposed the institution of "special schools

[81] See F. Enriques to F. Klein, 10 January 1905, in Giacardi 2012, p. 263, and now also in Luciano, Roero 2012.

[82] F. Klein, *Elementarmathematik vom höheren Standpunkte aus, I Arithmetik, Algebra, Analysis, II Geometrie, III PräzisionsundApproximationsmathematik*. Berlin, Springer, 1925–1933 (1st ed. 1908–1909), II, pp. 245–250.

[83] F. Enriques 1907, *Sulla preparazione degli insegnanti di Scienze*, pp. 69–78 in *Quinto Congresso nazionale degli insegnanti delle scuole medie. Bologna, 25-26-27-28 settembre 1906*, Atti. Pistoia, Tip. Sinibuldiana, p. 78.

[84] Quoted in Simili 2000, p. 114.

of Application" which were to group together professional teaching aimed at a specific career, the polytechnic schools for engineers and the polyclinical schools for physicians, and the Scuole di Magistero for the training of teachers.

– In 1907 Enriques founded, with Eugenio Rignano, the *Rivista di Scienza* (from 1910 *Scientia*), "an international organ of scientific synthesis", aimed at fighting excessive specialisation in the field of science and putting an end to the hegemony of literary and historic studies;[85] his imprint is particularly evident in the early years.

– In 1911 he organised the fourth International Congress of Philosophy in Bologna.

– From 1908 to 1920 he was Italian delegate to the ICMI, together with Castelnuovo and Vailati, who was substituted, after his death, by Gaetano Scorza.

– From 1912 to 1915 he was President of the National Association of University Professors.

5.3. Institutional and editorial initiatives for teacher training

Enriques' efforts and commitment to the training of teachers, and thus more generally to the improvement of mathematics education in secondary schools, are truly remarkable at both the institutional and the editorial levels.

– From 1919 to 1932 Enriques was president of the Mathesis Association. His desire to open up to other sciences is evident in the new charter for the association, which, on 7 May 1922, welcomed teachers of physics into its ranks, and led the society to assume a new name: *Società italiana di scienze fisiche e matematiche "Mathesis"*. Under the leadership of Enriques, the number of members grew from 775 in 1920 to more than 1,200 in 1924. Then, since in 1921 the *Periodico di Matematiche* had gone back to being the association's publishing venue, he assumed its direction (1921–1937, 1946)[86], giving it a strong imprint.

– From 1903 to his death he wrote many secondary school textbooks (for geometry, algebra, trigonometry, and infinitesimal calculus), in which he translates his vision of mathematics teaching into practice, thanks also to the valuable collaboration of Amaldi.

– In 1923 he founded the National Institute for the History of Sciences and the following year the University School for the History of the Sciences connected to it with the threefold aim of giving an impulse to historical research, of achieving his ideal of a scientific *humanitas*, and of training mathematics teachers.

[85] See Linguerri 2005.

[86] From 1921 to 1934 he was co-director with Giulio Lazzeri. From 1938 to 1943 Enriques' name does not appear on the title page as a consequence of the racial laws passed in 1938. In 1946 he was co-director with Oscar Chisini.

– Enriques also fostered important and successful publishing projects. First of all he broadened the 1900 work on elementary geometry, publishing his *Questioni riguardanti le matematiche elementari* (2nd ed. 1912–1914, 3rd ed. 1924–1927), a collective work whose declared aim was to contribute to the scientific and didactic training of mathematics teachers, and offer a broader vision of the theories and historical progress of mathematical science, but also to "illuminate the most elevated research as well as open the fruitful field of historical investigation to a larger number of scholars".[87] In carrying out this undertaking, he involved his friends and colleagues and various secondary school teachers.
Less well known is the book series begun in 1925 by Enriques, *Per la storia e la filosofia delle matematiche*, for which he also drew on the collaboration of secondary school teachers. The idea behind the series came, as he himself wrote, "from teaching experience at the Scuola di Magistero";[88] the series' target audience was not only teachers, but also secondary school students and the educated public in general. In the choice of the subjects Enriques particularly favoured translations with commentaries, often accompanied by historical notes, of works by important authors of the past (Euclid, Archimedes, Bombelli, Newton, Dedekind, etc.) which might be of relevance to mathematics teaching. In fact he believed that:

> "The training of mathematics teachers who are capable of carrying out their educational responsibilities requires, generally speaking, that they understand science not only in its static aspect, but also in its developing state; and thus that they learn from history to reflect on the genesis of the ideas, and on the other hand, take an active interest in research."[89]

Each of these initiatives deserves to be presented in detail, but I will limit myself to mentioning the change in style that Enriques impressed on the *Periodico di matematiche* because it so aptly exemplifies his cultural project.[90]
The imprint of the fourth series, which began with the 1921 volume, is exquisitely Enriques', as evident starting with the title – *Periodico di Matematiche. Storia - Didattica - Filosofia* – and continuing with the introductory sentence that appears on the inside of the front cover of each issue of the journal:

> "The *Periodico* publishes above all articles regarding elementary mathematics in a broad sense, and others that tend towards a wider comprehension of the spirit of mathematics. It also contains reports on movements in mathematics abroad, notes on bibliographies and treatises, miscellany (problems, games, paradoxes, etc.) as well as news of a professional

[87] F. Enriques, *Prefazione*, in *Questioni riguardanti le matematiche elementari. Raccolte e coordinate da Federigo Enriques*. Bologna, Zanichelli 1924–1927 (Anastatic rpt. Bologna, Zanichelli 1983).
[88] F. Enriques, *Gli Elementi d'Euclide e la critica antica e moderna (Libri I–IV)*, Rome. Alberto Stock, 1925, p. 7.
[89] Enriques, *Le matematiche nella storia e nella cultura*, cit. p. 190.
[90] For the other initiatives, see Giacardi 2012.

> nature, and finally, the Proceedings of the Italian Mathematical Society Mathesis".

According to Enriques' project, the *Periodico* was intended to disseminate the idea of mathematics as an integral part of philosophical culture, an idea he had always supported, as well as to fill the gap that existed in scientific education at that time in Italy. For this reason he gave ample space to questions of methodology and philosophy, to elementary mathematics from an advanced standpoint, to physics and to history of mathematics and science, availing himself of the collaboration of mathematicians, physicists and historians of science (Ugo Cassina, Giulio Vivanti, Enrico Persico, Enrico Fermi, Ettore Bortolotti, Gino Loria, Amedeo Agostini, etc.). Enriques also encouraged an active collaboration on the part of mathematicians and young people "provided with a solid scientific preparation", but also on the part of "teachers who wished to offer the contribution of their experience".[91]
In the letter to the readers that opened the 1921 issue, he presented a working program for the journal, which was at the same time a working program for teachers. The cardinal points are: teachers should study the science that they are teaching in depth from various points of view, so as to master it from new and higher points of view, and thus make evident the connections between elementary mathematics and higher mathematics; use the history of science in the attempt to attain, not so much erudite knowledge as a dynamic consideration of concepts and theories, through which students can recognise the unity of thought; bring out the relationships between mathematics and the other sciences, and physics in particular, in order to offer a broader vision of science and of the aims and significance of the many different kinds of research.[92]
This open letter was followed by Enriques' famous article, "Insegnamento dinamico", which is almost a manifesto of his working program and of his particular vision of mathematics education: active teaching, Socratic method, learning as discovery, the right balance between intuition and logic, the importance of error, the historic view of problems, the connections between mathematics and physics, elementary mathematics from an advanced standpoint, and the educational value of mathematics.[93]
In his 1931 preface to the index of the first ten years of the second series of the *Periodico di matematiche*, Enriques underlined with pride the role played by the journal in teacher training:

> "No other journal of this sort, in no other country in the world, has been able to realise a program that is as lofty and attuned to the exigencies of education and culture of teachers of middle schools."[94]

Almost all of these last initiatives of Enriques, in the final analysis aimed at creating a scientific humanism, took place after the Gentile Reform, when the Fas-

[91] Enriques, *Ai lettori*, Periodico di matematiche, 4, 1, 1921, pp. 1–5.
[92] Enriques, *Ai lettori*, cit., pp. 3–4.
[93] Enriques, *Insegnamento dinamico*, cit.; see also the new edition accompanied by essays by F. Ghione and M. Moretti published by the Centro Studi Enriques: La Spezia, Agorà 2003.
[94] See "Indice generale Serie IV - Volumi I a X – Anni MCMXX-MCMXXX", *Periodico di matematiche*, (4) 11, 1931, pp. 3–21.

cist period had well begun. In 1923 Giovanni Gentile, minister for Education in Italy's Fascist government, carried out a full and organic reform of the school system in accordance with the pedagogical and philosophical theories he had been developing since the beginning of the 1900s. As is well known, according this reform, the secondary school system was dominated by the classical-humanistic branch, which was designed for the ruling classes and considered absolutely superior to the technical-scientific branch. The principles of Fascism and the neo-idealist ideology were opposed to the widespread diffusion of scientific culture and, above all, to its interaction with other sectors of culture. Humanistic disciplines were to form the main cultural axis of national life and, in particular, of education. This point of view was, of course, opposed to the scientific *humanitas* to which Enriques aspired. As president of the Mathesis Association, he engaged in intense negotiation with Gentile, both before and after the law on secondary education was enacted, in the hope of avoiding the devaluation of science teaching. However, the pleas of the Mathesis fell on deaf ears.

Unlike Volterra and Castelnuovo, who were in absolute opposition to the Gentile Reform, Enriques assumed and maintained a conciliatory position. In fact, his ideal was to achieve a fusion between "scientific knowledge" and "humanistic idealism" in a "superior awareness of the universality of thought".[95] However, he was opposed to all forms of nationalistic isolation, as is shown, for example, by the organisation of the international meeting entitled "Settimana della Scuola di Storia delle Scienze" (Rome, 15-22 April 1935). Among the participants, in addition to lecturers at his School for the History of the Sciences (Roberto Almagià, Silvestro Baglioni, Giuseppe Montalenti, Giovanni Vacca), there were Castelnuovo, Enrico Bompiani and Giuseppe Armellini and twenty-six members from London's Unity History School as well as scholars from other European countries, including the Belgian Paul Libois, who would draw various aspects of his own vision of mathematics teaching from Enriques, and the French historian Hélène Metzger, who shared Enriques' unitary concept of science. The topics addressed ranged from philosophy to the history of physics, astronomy, biology and technology, and the debate was lively, as can be seen from the detailed summary of the week's activity written by Metzger herself.[96]

During the same period Enriques also participated in the meetings (Paris, Vienna, Berlin) and congresses (Heidelberg, 1927; Barcelona, 1929; Paris, 1933; Budapest, 1934; Zurich, 1938) of the *Fédération internationale des Unions intellectuelles*, in addition to various other international congresses of philosophy, history of philosophy and philosophy of science: it was no coincidence that Enriques remained in contact with the *Fédération* whose aim was to promote international cultural exchange. He also directed two sections – *Philosophie et histoire de la pensée scientifique and Histoire de la pensée scientifique* – of the book series *Actualités scientifiques et industrielles* published by Hermann in Paris. Between 1934 and 1939 eight volumes were published in the first series, with the collaboration of Metzger, Ferdinand Gonseth and Castelnuovo, and six in the second series, written in collaboration with Giorgio de Santillana.

[95] Enriques, *Il significato umanistico della scienza nella cultura nazionale*, cit. p. 4. See also Israel 1984, Guerraggio, Nastasi 1993, Pompeo Faracovi 2006

[96] See Metzger 1935.

Up the end in 1946 Enriques fought his battle for a scientific humanitas and was involved in teacher training, which he believed to be the crucial element for the formation of good schools and one of the channels for achieving his cultural project.

6. Francesco Severi: politics and education.

Graduating with Segre in 1900, Severi was, as is well known, a top level mathematician who made very significant contributions in the field of algebraic geometry as well as various other areas in mathematics, which earned him numerous prizes, recognitions and prestigious positions over the course of his career.

Two factors are of prime importance for fully understanding Severi's stance with regard to education: his singular political path, and his relationship, first of collaboration and then of conflict, with Enriques.

Severi was a socialist during the period he was in Padua; as rector in Rome, he resigned after the murder of Giacomo Matteotti; he was a signer of Benedetto Croce's Manifesto of the Anti-Fascist Intellectuals; and a supporter of those who opposed the fascistization of the University of Rome. Nevertheless, following his nomination to the Accademia d'Italia in the spring of 1929, he supported Fascism without reserve and later had no compunction about using the racial laws to assume absolute control over Italian mathematics.

His collaboration with Enriques began right after his degree, intensified during the period in which Severi was Enriques' assistant in Bologna, and reached its peak in their work on hyperelliptic surfaces, which was awarded the Prix Bordin of the Académie des Sciences in Paris (1907). In the years that followed their relationship was increasingly marked by divergences on scientific, academic, editorial and cultural levels.[97]

6.1. Enriques' influence and successive rivalries

To be sure, the influence of Enriques is one of the principal factors underlying Severi's interests in mathematical epistemology and teaching. To see this we need only look at the writings and events of the period from 1902 to 1920. In 1906 Severi published his *Complementi di geometria proiettiva* (1906) as an integration to Enriques' *Lezioni di geometria proiettiva* (1903). The two textbooks were born in symbiosis, and Severi accepted the epistemological and didactic vision of his mentor. Between 1906 and 1920 he wrote various articles and reviews[98] which also demonstrate an acceptance

[97] Cf. Brigaglia, Ciliberto 1995, pp. 24–32 and 36–41; the essays of Brigaglia, Ciliberto and Vesentini, in Pompeo Faracovi 2004.

[98] 1903, *Estensione e limiti dell'insegnamento della matematica, in ciascuno dei due gradi, inferiore e superiore, delle Scuole Medie*, Il Bollettino di Matematica, 2, pp. 50–56 (with F. Enriques and A. Conti); 1906, Review of *F. Enriques, "Problemi della scienza"*, Rivista di filosofia e scienze affini, 8, 2, pp. 527–541; 1908, Review of G. Loria, *"Il passato e il presente delle principali teorie geometriche"*, Rivista di Scienza, 4, pp. 376–378; 1910, *Ipotesi e realtà nelle scienze geometriche*, in *Atti della Società Italiana per il Progresso delle Scienze*, 3, pp. 191–217 (also in Scientia, pp. 1-29 and the French translation in Suppl. pp. 3-32); 1911, *La nostra scuola*, Padova; 1914, *Razionalismo e spiritualismo*, Conferenze e prolusioni, 10, pp. 181–189; 1919, *La matematica*, Energie Nuove, 9, pp. 196–199; 1920, *L'istruzione professionale*, in *Atti del Congresso degli Agricoltori e Bonificatori*, Padova.

of many of Enriques' methodological assumptions: knowledge proceeds by successive approximations; geometry is seen as a part of physics; the historical and psychological genesis of mathematical concepts; the importance of analogies and induction in discovery; the use of an experimental, intuitive approach in mathematics teaching.
In particular, in the 1914 article entitled "Razionalismo e spiritualismo" Severi sided with Enriques against the idealism of Croce, proclaiming the cognitive and aesthetic value of science and illustrating the harmful consequences of the "movement against science" on the levels of society and education.[99]

Severi's burning ambition to occupy top level positions within the mathematical and academic communities inevitably led to his first clashes with Enriques on scientific, academic and personal planes. He knew how driven he was; he himself said, "My will is tenacious to the point of obstinacy".[100]
When, in 1909, he became president of the Mathesis Association, Severi attempted to insert himself into the work of the Italian subcommission of the ICMI, whose three delegates at the time, nominated directly by ICMI Central Committee, were, as we have said, Castelnuovo, Enriques and Vailati. In fact, the Mathesis Association was not officially part of the delegation. To reach his objective, Severi sought the support of Volterra, and even suggested that Vailati should be encouraged to resign: "...poor Vailati, afflicted as he is by his long illness, might do well to step down ...and then much could be put to rights by having a replacement elected by the Mathesis".[101] His attempts to impose himself were not successful because Enriques and Castelnuovo believed that it was important that the subcommission, while collaborating with the Mathesis, maintain its "freedom to act" and not be obliged to conform to the directives of the Association.[102]
This first setback was followed by another. During his term as president, Severi sent repeated requests (January 1909, February and April 1910) to the different Ministers for Education at the time asking them to consider the proposals of the Mathesis regarding the reform of the Scuole di Magistero for teacher training, the abolition of the choice between Greek and mathematics beginning in the second year of liceo that had been introduced by the Orlando Decree of 1904, and the reinstatement of the written exam in mathematics for all categories of schools. Severi was able to obtain from the Minister only a few general promises, and in all likelihood these setbacks drove him to look for different ways to achieve his ends and impose his power on the mathematical and academic communities. Thus on 6 November 1901 he announced his resignation and that of the Mathesis executive committee:

> "And we intend to communicate our decision to the largest daily newspapers, so that public opinion will pause, at least for a moment, to consider whether the slight regard in which cultural Societies, such as ours, are held by executive power, constitutes the most suitable means for stimulating that disinterested attachment to Education, which, despite everything,

[99] F. Severi, *Razionalismo e spiritualismo*. Conferenze e prolusioni, 10, 1914, pp. 181–189, at p. 187.
[100] F. Severi, *Confidenze*, La scienza per i giovani, 1952, II, pp. 65–69, at p. 69.
[101] F. Severi to V. Volterra, Padova, 20 April 1909 in Nastasi 2004, p. 180.
[102] See the letters of Severi to Volterra in Nastasi 2004, pp. 176–181.

teachers still show themselves to hold."[103]

In any case, Severi deserves the credit for having put his finger, during his brief term as president, on the two main weaknesses of the Mathesis, calling for, on one hand, the reform of the *Bollettino della Mathesis*, which was supposed to be transformed from a simple administrative tool into a journal with articles about science and education, and on the other hand, a strengthening of the Association's congresses, which were to offer rich programs and, above all, fighting absenteeism.[104] His wishes would be carried out by the presidents who succeeded him, first Castelnuovo and then Enriques.

After Croce's sharp attack on his article "Razionalismo e spiritualismo",[105] Severi began to distance himself from Enriques, which led to scientific and cultural battles to dominate Italian mathematics. These, as has been said, led to a genuine "pursuit" on scientific, academic, educational, editorial and cultural planes.[106] In 1921 Severi brought to light an error in an article of Enriques, leading to a polemic that would last over twenty years.[107] That same year, supported by Tullio Levi Civita, Severi had the better of Enriques for the transfer to Rome to the chair of algebraic analysis left vacant by Alberto Tonelli. Enriques would assume the chair in higher geometry in 1923, thanks only to Castelnuovo's renunciation of it.[108]

Quick to understand the mechanisms of political power and exploit them to his own advantage, with his nomination to the Accademia d'Italia in 1929,[109] Severi went, as mentioned above, from being an anti-Fascist to being a fervent Fascist. In 1929-1931 he had no qualms about collaborating on the draft of a new form of oath of loyalty to the Fascist party.[110] He then began to cooperate in the process of the fascistization of culture, contributing to widen that breach between Italian mathematicians and the international mathematics community which was one of the reasons for the weakening of mathematics research in Italy that ensued. When he later became conscious of this fact, he attempted to halt the process of decline by creating in 1939 of the Istituto di Alta matematica (Institute for Higher Mathematics).[111]
On this aspect of Severi's personality, Francesco Tricomi wrote:

> "Severi ... wanted to be (and to a certain extent, was) the 'godfather' of Italian mathematics during the Fascist period. We in any case have the consolation of knowing that — while, as a rule, totalitarian regimes put

[103] See *Dimissioni del CD*, Bollettino della "Mathesis", 2, 1910, p. 90.

[104] See *Sezione veneta, Adunanza del 20 maggio*, Bollettino della Mathesis, 1, 1909, pp. 31–32, and *Programma del prossimo Congresso sociale*, Ibid., pp. 51–52.

[105] B. Croce, *Se parlassero di matematica?*, La Critica. Rivista di letteratura, storia e filosofia, XII, 1914, pp. 79–80.

[106] See Faracovi 2004, especially the essays of Vesentini, Ciliberto, Brigaglia, Bolondi, Faracovi and Linguerri.

[107] Brigaglia 2004, pp. 66–77, Ciliberto 2004, pp. 44–49.

[108] See T. Nastasi 2010, Appendice 2, *Il trasferimento di Enriques a Roma*.

[109] Enriques' name was included on the early list of candidates of scientific disciplines but was stricken at the last moment; see Goodstein 1984, p. 294.

[110] F. Severi to G. Gentile, Barcelona, 15 February 1929, in Guerraggio, Nastasi 1993, pp. 211–213.

[111] On the effects of his activities on Italian research in mathematics, see for example, Israel 1984 §5–6; Guerraggio, Nastasi, 1993; Israel 2010, Chap. 6.

> the worst elements in positions of control, only because they are violent or subservient or both — in the case of Severi, the man was, from a scientific point of view, irreproachable".[112]

The "Severi case" has been amply studied by historians, so here I will only mention Severi's open opposition to Enriques. He refused to collaborate with the *Enciclopedia italiana* on the mathematics section, of which Enriques was director, writing: "with a man such as Enriques, ...I can no longer have anything in common, much less a relationship akin to subordination."[113] He opposed the request that university chairs be established for history of science, presented by Enriques to the Accademia dei Lincei in 1938.[114] That same year Italy's shameful racial laws were put into effect, and Severi unhesitatingly exploited them in order to rise to a position of absolute predominance in Italian mathematics. In fact, when Enriques was dismissed from the University because of the racial laws against Jews, he immediately transferred to the chair of higher geometry held by Enriques, and in February 1939 he assumed direction of the University School for the History of the Sciences created by Enriques, leading finally to its closure. As president of the Vallecchi publishing house in Florence, he took advantage of the circular issued by Minister of Education Giuseppe Bottai in August 1938, which ordered school principals to eliminate from use all textbooks written by Jewish authors, to replace the geometry textbooks for secondary schools by Enriques and Amaldi with his own textbooks, published by Vallecchi.[115]

Severi's opinions regarding the Gentile Reform were in many respects similar to those of Enriques: he was convinced of the superiority of the *ginnasio-liceo*,[116] he was in favour of combining mathematics and physics but held that too few hours were dedicated to mathematics, and that the number of hours assigned to teachers (22) was too heavy.[117] There were however points where their opinions differed: Severi tended to share the nationalistic and autarchic vision of scientific research,[118] while Enriques instead observed:

> "In scientific discovery there is ...a universal value that transcends the person of the discoverer, and also the forma mentis that he may have received from his people. ...greatness and decline of culture alternate to the

[112] Tricomi 1967, p. 55.

[113] F. Severi to G. Gentile, Arezzo 24 May 1928, in Guerraggio Nastasi 1993, pp. 209-210; see also G. Bolondi, *Enriques, Severi, l'Enciclopedia Italiana e le istituzioni culturali*, in Pompeo Faracovi 2004, pp. 79–106.

[114] Enriques, *L'importanza della storia del pensiero scientifico nella cultura nazionale*, cit.

[115] S. Linguerri, *Federigo Enriques e Francesco Severi: una concorrenza editoriale*, in Pompeo Faracovi 2004, pp. 151-154.

[116] *Riunione straordinaria promossa dal consiglio direttivo, Roma 11 febbraio 1923*, Periodico di Matematiche, 1923, pp. 156–157.

[117] See F. Severi, *L'insegnamento della geometria nei suoi rapporti colla riforma*, Annali dell'istruzione media, 3, 1927, pp. 108–116, at p. 116.

[118] See, for example, F. Severi, *Scienza pura e applicazioni della scienza*, in *Atti del I Congresso dell'Unione Matematica Italiana*, Zanichelli, Bologna, 1937, pp. 13–25; also in Scienza e tecnica, 1, 1937, n. 4, pp. 8-1-89, at p. 89; F. Severi, *Interventi al Convegno di Padova per l'istruzione media, classica, scientifica e magistrale*, Scuola e cultura,1939, pp. 62–65, at p. 65.

> extent that the exchange of ideas with other peoples, near or far, in space and time, either grow and intensify, or oppositely, weaken."[119]

How Severi adapted himself to Fascist directives can also be seen in his *Curriculum vitae*, where he states that he "had contributed also with his textbooks concerning the most elementary fields of mathematics, to renovate teaching methods" in middle schools, "adapting them to new lines of knowledge and new pedagogical needs determined by Fascism".[120] On the other hand, Gentile, in the preface to one of Severi's geometry textbooks wrote:

> "I am pleased to see that books such as these by Prof. Severi are beginning to be published for the study of mathematics in middle schools.
> The new Italian school must be an active school, one which sets, at all levels and in all forms of teaching, the student's spiritual strengths into motion, allowing him to feel the fatigue and joy of understanding for himself, of discovering for himself and acquiring his own truth ... And to me these books seem to correspond wonderfully to our desire that these subjects as well ... be presented in the most suitable form for beginners: the heuristic form of the concept arrived at by means of intuitions that are concrete, evident and attractive."[121]

Furthermore, when in 1939 the Grand Council of Fascism approved the twenty-nine declarations contained in the *Carta della Scuola* (School Charter) presented by Minister of Education Bottai with the aim of further fascistizing Italian schools, Severi declared that he agreed "to every single part of it".[122]

6.2. Mathematics teaching: methodological assumptions and their effects on textbooks

This said, the cornerstones of Severi's methodological and pedagogical vision were nevertheless very close indeed to those of Enriques, although the epistemological considerations upon which they were founded were not as broad and detailed:

- secondary school must have an essential formative aim and a "frank humanistic basis"; to these ends mathematics plays an important role because it trains the faculties of intuition and abstraction and develops an aptitude for "observing, abstracting, and deducing";[123]

[119] F. Enriques, *L'Italia nella collaborazione universale della cultura*, Nuova Antologia, s. 7, 247, 1926, pp. 129–134, at p. 132, 133.

[120] F. Severi, *Curriculum vitae* (1938-XVI), in `http://dm.unife.it/matematicainsieme/riforma_gentile/pdf/Gentile09.pdf`.

[121] F. Severi, *Elementi di geometria pel ginnasio e pel corso inferiore dell'istituto tecnico, Volume I*, Firenze, Vallecchi, 1926, p. V.

[122] Severi, *Interventi al Convegno di Padova*, cit. p. 63.

[123] F. Severi, *Relazione al Convegno di Firenze per l'istruzione classica scientifica e magistrale*, Scuola e cultura, 1940, pp. 70–73, at p. 72–73.

– humanism must not be disjoined from scientific thought, in fact true humanism is integral by nature[124] – thus it is necessary to transmit to the student a unitary vision of culture, and scientific, historic, literary and philosophic teaching must be "maintained in the same plane";[125]

– mathematics teaching must have an intuitive character in lower middle schools and a rational character in upper middle schools, proceeding by "successive approximations" from the concrete to the abstract, and allowing time for the ideas to "filter slowly through the brains, if it is desired that they leave traces that are useful and lasting".[126] In any case, in teaching precedence must be given to intuition because it develops in a way that is natural and direct, as a "synthesis of sensations, observations and experiences", almost without any wilful effort at attention on the student's part,[127] and because only intuition provides the raw material to the logical machine:

> "[students are] taught to reason ... by reasoning well; not by dissecting the reasoning";[128]
> "It is necessary to take middle school teaching of mathematics back to its practical and intuitive origins; and this not only for practical reasons (which in middle school could have no prevailing weight), but above all precisely for the educational goals of secondary studies;[129]

– it is important to use "the utmost parsimony in formulating programs, reducing them for each discipline to things which are truly essential and which have unquestionable educational value".[130] In particular, Severi suggests abandoning the cyclical method by which subjects already treated in an intuitive way in middle schools are repeated in a rationally developed way in secondary schools, and to "bring teaching closer to the current state of science";[131]

– it is useful for teachers to link mathematics teaching to that of physics in order to "give new impetus to their own teaching by means of continuous and fruitful contact with the real world";[132]

– the teacher must play a central role in guiding the student in learning:

> "Having discovered the main path [to learning], it is necessary to travel it anew, and to clear away the difficulties that are too serious

[124] F. Severi, *Relazione al Convegno di Messina per l'istruzione media, classica, scientifica e magistrale*, Scuola e cultura, 1940, pp. 136–138, at p. 137.
[125] Severi, *Relazione al Convegno di Firenze*, cit., p. 70.
[126] F. Severi, *La matematica*, Energie nove, II serie, 9, 1919, pp. 196–199, at p. 197; see also F. Severi, *Didattica della matematica*, Enciclopedia delle Enciclopedie: Pedagogia, Roma, Formiggini, 1931, pp. 362–370, at p. 365.
[127] Ibid., p. 198.
[128] Severi, *Didattica della matematica*, cit., p. 368.
[129] Ibid., p. 368.
[130] Severi, *Relazione al Convegno di Messina*, cit. p. 138.
[131] Severi, *Relazione al Convegno di Firenze*, cit. p. 72, 73.
[132] Severi, *Didattica della matematica*, cit., p. 365.

> for non-experts, so that the student can travel them along with us, following us, without excessive effort, in the process of constructing knowledge";[133]

– it is necessary to stimulate "the youthful desire for conquest", to involve the students in the process of constructing knowledge and exhort them to acquire mathematical truths for themselves: "allowing them to find everything nice and ready, does them no good";[134]

– the history of science can play a significant educational role: Severi, like Enriques, believed that in order to facilitate students' comprehension of certain mathematical concepts it is useful to take their historical origins as a point of departure,[135] and he himself used history in his lessons at university[136] as well as in the courses of specialisation:

> "don't forget the masters, because an ingenious idea is worth more in creative power than all of its consequences. And in order to follow the thought of the masters it is necessary to not distance ourselves from historical development of the ideas and from that troublesome but indispensable instrument, the bibliography."[137]

A brief overview of the way in which Severi conceived mathematics teaching appears in the entry "Didattica della matematica" that he wrote for the *Enciclopedia delle Enciclopedie* (1931), which includes an historical excursus about the teaching of this discipline in Italy that goes from the use of the textbooks by Legendre and Bertrand at the beginning of the nineteenth century up to the Gentile Reform.

How this vision of teaching is translated into practical terms emerges above all from the textbooks for lower and upper secondary schools, which constitute Severi's most important and lasting legacy regarding secondary teaching. Beginning in 1926, he directed the book series entitled *Collezione di testi di matematica per le scuole medie* for the Vallecchi publishing house in Florence. The series included textbooks for geometry, arithmetic, algebra (with trigonometry, financial mathematics and infinitesimal analysis), which were often written in collaboration with two teachers, his niece Maria Mascalchi[138] and Umberto Bini.[139] The distinguishing features of the books in this

[133] F. Severi, *Elementi di geometria pei licei e pel corso superiore dell'istituto tecnico, Volume II*, Firenze, Vallecchi, 1927, p. V.

[134] Ibid.

[135] See for example Severi, *Didattica della matematica*, cit., pp. 362-370.

[136] See R. Migliari, *L'insegnamento della Geometria Descrittiva e delle sue applicazioni*, in *La Facoltà di Architettura di Roma "La Sapienza" dalle origini al duemila. Discipline, docenti, studenti*, a cura di Vittorio Franchetti Pardo, Roma, Edizioni Gangemi, 2001, pp. 279–282.

[137] F. Severi, *Del teorema di Riemann-Roch per curve, superficie e varietà. Le origini storiche e lo stato attuale*. Varenna, CIME 1955, Roma, Istituto matematico dell'Università, 1955, p. 38.

[138] Maria Mascalchi (1902-1976), with a degree in mathematics at the University of Turin in 1923, and in physics in 1931, obtained in 1928 the chair in mathematics and physics at the Liceo classico d'Azeglio in Turin. See Archivio Storico of the University of Turin and the Archivio storico of the Liceo classico d'Azeglio in Turin.

[139] Umberto Bini taught at the R. Liceo scientifico in Rome.

series are the use of an intuitive approach, but with due attention to rational aspects, suitably arranged according to school level and type of school, brevity of treatment, mentions of history of mathematics, questions to facilitate learning, good exercises, clarity, precision and conciseness. In particular, the textbook entitled *Elementi di geometria*, adapted for the various types of schools, is distinguished by its particular approach to the principal topics of geometry (congruence, equivalence, the parallel theory, theory of proportions), as well as for the methodological framework dictated by the concern that the student not overlook the intuitive underpinnings of each notion introduced.

Rather than going into detail regarding the individual textbooks, I will mention only the numerous reflections of Severi scattered throughout his writings regarding the criteria to be respected in order to produce a good mathematical textbook.[140]
First of all, with regard to the use of textbooks, Severi observes that while in primary school teaching of geometry and arithmetic must be essentially oral, and "the importance of the textbooks is minimal",[141] "in secondary schools the advice to follow the texts, without the deplorable system ... of taking notes, must be strictly respected".[142] In general, "the exposition of the subject must ... allow it to be assimilated by mediocre minds and yet encompass a more hidden meaning, which induces better minds to more profitable meditation";[143] the treatment must be rich in intuitive observations, and must make "clearly distinguishable, for those who have the capacity, that which is taken from intuition and that which must be deduced".[144]

With regard to the kind of language to be adopted, it is necessary that "particular care be taken, both from the point of view of correctness, sobriety, correspondence between word and idea, and that of simplicity".[145]

Concerning proof, according to Severi this must be presented at the first stage as equivalent "to a reduction to the evidence", and the postulates must appear to the students as "explicit expressions, which could have remained unstated, of intuitive facts":[146] it is not necessary that the purely logical function of these be understood. Rigour can be arrived at gradually, taking care to give greater importance to "rigour in substance" – in line with which the framework of the treatment must appear to be "impeccable from a rational point of view" – rather than to "formal rigour".[147]

---

[140] See Giacardi, L., Tealdi, A., *Francesco Severi and mathematics teaching in secondary schools. Science, politics and schools in the first half of the twentieth century*, in "Dig where you stand" 3. Proceeedings of the Third International Conference on the History of Mathematics Education, K. Bjarnadóttir, F. Furinghetti, J. Prytz, G. Schubring (Eds.), Uppsala: Uppsala University, 2015, pp. 187-202.

[141] Severi, *Didattica della matematica*, cit., pp. 368–369.

[142] Severi, *Relazione al Convegno di Firenze*, cit. p. 71.

[143] F. Severi, *Elementi di geometria I pel ginnasio e pel corso inferiore dell'istituto tecnico*, edizione completa, 4° ristampa, Firenze, Vallecchi, 1933, p. IX.

[144] Severi, *Didattica della matematica*, cit., p. 368.

[145] F. Severi, M. Mascalchi, *Nozioni di Aritmetica pratica, con cenni storici per il* 1° *e* 2° *anno della Scuola media*, (con M. Mascalchi), Firenze, Vallecchi, 1941, p. 1.

[146] F. Severi *Elementi di geometria pel ginnasio e pel corso inferiore dell'istituto tecnico*, Firenze, Vallecchi 1926, pp. VIII, IX.

[147] Ibid., p. VIII.

Another point which Severi particularly emphasises is the importance of having opportune explanations precede the definitions of the mathematical objects:

> "I have taken the most scrupulous care to avoid definitions *ex abrupto*. These are most irksome. I do not give definitions without an appropriate prelude ...And I have no fear of being verbose. ...The time given to a good understanding of the meaning of a definition is ...excellently spent, always ...He [the student] must construct the definitions himself, beginning with the common sense notions that he possesses".[148]

Finally, Severi was a fervent supporter of the need for brevity of treatment, stripping it of anything that is not essential to the comprehension of the structure of a mathematical theory, with the aim of both "reducing the burden of the students, without damaging the educational function of the teaching of mathematics, and geometry in particular",[149] and of making room for more modern topics. Nevertheless, he emphasises that "simplicity and clarity deriving from implied statements, that are neither educationally nor scientifically honest, are not what I aspire to, and nothing could induce me to introduce such an approach into my work".[150]

Conclusions

From our examination of the commitment to questions regarding mathematics teaching of these four eminent exponents of the Italian School of algebraic geometry emerges a core set of shared assumptions whose roots lie in their common way of conceiving mathematical research, and which constitute an additional indicator of the appropriateness of the term "School" in speaking of the Italian geometers. Using the word "School", we are referring both to a group of researchers trained by the same *maestri*, from whom they draw topics of investigation, methodologies, approaches to research and a particular scientific style, and a place where talents are developed and contacts made, as well as an environment, as we have tried to show, in which a common vision of the transmission of knowledge matures, while still considering the opportune *distinguo*.

Thus, if we attempt to draw a conclusion from their multifaceted activities and provide a comprehensive overview, we first of all observe that the common vision of scientific research, and the influence of Klein, so clearly documented above all in Segre, Enriques and Castelnuovo – who were able to meet him personally and work for some time alongside him – led these mathematicians to share the following pedagogical assumptions:

– the attribution of an educational value to mathematics, in the hope of creating a scientific *humanitas* (an integral humanism);

[148] Severi, *Elementi di geometria I pel ginnasio e pel corso inferiore dell'istituto tecnico*, 1933, cit. p. X.
[149] F. Severi, *Geometria, Volume I*, Firenze, Vallecchi, 1934, p. V.
[150] Ibid., p. VII.

– using to the best of their advantage the faculty of intuition and the heuristic procedures in teaching;

– aiming at rigour in substance (large-scale logic), rather than formal rigour (small-scale logic, microscope, etc.);

– the establishment of connections between mathematics and other sciences, and between mathematics and applications;

– the attribution of importance to the history of mathematics in teaching and in research.

These were the assumptions that directed their activities in education, in spite of the fact that their motivations and even the strategies they employed sometimes followed different channels.

For Segre it was above all the intimate connection that he saw between teaching and research that led him to become personally involved in teacher training, and at the same time present to his university students topics that were useful for teaching. Instead, Castelnuovo's motivation was mainly social, because he believed it was important to train people to be capable of understanding the reality in which they live and work in order to improve it. His idea for the *liceo moderno* grew out of his belief in a cultured democracy, one capable of providing the basis of a modern nation. The channels he used to make his ideas concrete were essentially three: involvement with national and international institutions – ICMI, the Mathesis Association, the Ministry of Education; teaching university courses; and directly involving teachers.

What led Enriques to become involved in problems of education and in secondary teaching were his strong philosophical, historical and interdisciplinary interests, especially the studies on the foundations of geometry. He adopted a range of strategies and, as we have seen, worked on different fronts: institutional, editorial (periodicals, book series, textbooks), and cultural. Further, he addressed his activities to different categories – secondary school teachers, researchers, philosophers, scientists, people of culture – asking for their cooperation. His direction of the *Periodico di Matematiche* is significant in this respect.

Severi's intellectual itinerary was of yet a different nature: his interest in problems concerning the secondary teaching of mathematics was inspired both by his relationship, first of collaboration and then of rivalry for leadership, with Enriques, and by political reasons. After his unsuccessful bid to insert himself into the ICMI, and the sparse results of his presidency of the Mathesis Association, Severi supported the school policies of the Fascist regime, while holding firmly to the pedagogical assumptions of the Italian School of algebraic geometry. His channel of choice for improving mathematics teaching was the publishing of schoolbooks, and as might be expected of such a great mathematician, he produced textbooks that were paragons of clarity, precision and conciseness.

To a much greater extent than Segre, Castelnuovo and Severi, and above all Enriques, took special advantage of journals to spread their point of view. Castelnuovo, during his presidency of the Mathesis Association, used the *Bollettino della Mathesis*

to further his agenda, inserting reports on the work of the Italian subcommission of the ICMI. Without relying on any particular periodical, Severi published his reflections on teaching in different journals aimed at diverse readerships: before 1923, *Rivista di Scienza, Energie Nuove, Rivista di filosofia e scienze affini*, and after 1923, *Annali dell'istruzione media, L'illustrazione italiana*, and above all *Scuola e cultura*, a magazine that was particularly supportive of the Fascist Regime. It was, however, especially Enriques who exploited periodicals to his advantage in spreading his idea of scientific *humanitas*: for the *Rivista di Scienza* (later named *Scientia*) he wrote no fewer than 22 articles, 63 reviews and 25 digests of magazines, and to the *Periodico di matematiche* he contributed 26 articles, 33 reviews and numerous other interventions in his role as president, covering topics in mathematics, philosophy, history of science, and education.

Finally, these mathematicians differed in their reactions to the Gentile Reform and to the problems of the devaluation of science and the autarchy touted by Fascism. We have seen that Castelnuovo always maintained an attitude of determined opposition, while Enriques, even while continuing to uphold the educational and cultural value of mathematics, maintained a conciliatory position. In fact, he agreed with Gentile on many points: he was convinced that among the various kinds of secondary schools, those which best performed the function of education were the *ginnasi-licei* (schools with an emphasis on the humanities); he conceived of knowledge as a personal conquest; he was in agreement with the need to fight encyclopaedism and he considered education to be the free and unfettered development of inner energy. Moreover, as we mentioned above, he did not want to renounce his idea of the fusion of scientific knowledge and humanistic idealism which was the basis of the cultural program he had dedicated his whole life. With regard to the ideological tendency of the Fascism to give pre-eminence to applied sciences as a means of solving problems arising from autarchy, Enriques never abandoned his idea of linking pure mathematics to its applications, even though this originated primarily from philosophical and didactic considerations. Moreover, he did not share the principle of autarchy, but was open and ready to engage in a dialogue with the international scientific community.

Instead, Severi more than once expressed his acceptance of the cultural directives of the Fascist Government, and only later became aware of the harm that scientific isolation could lead to. In spite of this, he always maintained the pre-eminence of pure science, which he believed to be a source of beauty and art, over applied science, without any concession made to its social function:

> "more often than not, ideas, which are the true engines of human society, descend from abstract regions to enliven the applications and provide their most useful and powerful orientations. Where abstract science is neglected, practice soon becomes arid and civilisation begins a rapid decline".[151]

What the legacy of the wide-ranging and diversified activities of these Italian

[151] Severi, *Scienza pura e applicazioni della scienza*, cit. p. 83, 84; see also Israel, Nurzia 1989, at pp. 139–143.

geometers has been for the development of mathematics teaching in Italy is a subject that has been examined up to now only sporadically, and is not the aim of this present paper. However, to me their intellectual bequest seems evident, and I would like to highlight three aspects in particular: the belief that it is important that mathematicians who are active in research be involved in problems related to teaching; the need to invest substantial resources in teacher training and to bring about a greater interaction between universities and secondary schools; and finally, the belief that it is important to develop an integrated humanism in schools, a goal to which the history of mathematics can make a valid and valuable contribution.

I am very grateful to Gabriella Viola for her invaluable help in finding some of Severi's textbooks and other documents concerning him, and to Antonio Salmeri for his generous aid in bibliographic research. Thank you also to the personnel of the Biblioteca "Giuseppe Peano" of the Department of Mathematics of the University of Torino, and to Kim Williams for linguistic advice.

References

[1] BOTTAZZINI U., CONTE A. AND GARIO P., *Riposte armonie. Lettere di Federigo Enriques a Guido Castelnuovo*, Bollati Boringhieri, Torino 1996.

[2] BRIGAGLIA A., *Due modi diversi di essere caposcuola*, in Pompeo Faracovi (2004), 51–77.

[3] BRIGAGLIA A., *Da Cremona a Castelnuovo: continuità e discontinuità nella visione della scuola*, in *Da Casati a Gentile. Momenti di storia dell'insegnamento secondario della matematica in Italia*, Giacardi, L. (Ed.), Centro Studi Enriques, 6, Lugano: Lumières Internationales, (2006), 159–179.

[4] BRIGAGLIA A. AND CILIBERTO C., *Italian Algebraic Geometry between the two World Wars*, Queen's Papers in Pure and Applied Mathematics, **100**, Kingston, Ontario: Queen's University (1995)

[5] CHISINI O., *Accanto a Federigo Enriques*, Periodico di matematiche 4, **25**, 2 (1947), 117–123.

[6] GARIO P., *Guido Castelnuovo e il problema della formazione dei docenti di matematica*, Rendiconti del Circolo matematico di Palermo, s. II, Suppl. 74 (2004), 103–121.

[7] GARIO P., *Quali corsi per il futuro insegnante? L'opera di Klein e la sua influenza in Italia*, Bollettino della Unione Matematica Italiana. La Matematica nella società e nella cultura, s.8, **IX-A**, (2006), 131–141.

[8] GIACARDI L., *Educare alla scoperta. Le lezioni di C. Segre alla Scuola di Magistero*, Bollettino dell'Unione Matematica Italiana, s.8, **VI-A**, (2003), 141–164.

[9] GIACARDI, L., *From Euclid as Textbook to the Giovanni Gentile Reform (1867-1923). Problems, Methods and Debates in Mathematics Teaching in Italy*, Paedagogica Historica. International Journal of the History of Education, **XVII**, (2006), 587–613.

[10] GIACARDI L., *Guido Castelnuovo*, in Furinghetti, F. and Giacardi, L. (Eds.) `http://www.icmihistory.unito.it/portrait/castelnuovo.php`., 2008.

[11] GIACARDI L., *The Italian School of Algebraic Geometry and Mathematics Teaching in Secondary Schools. Methodological Approaches, Institutional and Publishing Initiatives*, International Journal for the History of Mathematics Education 5, **1**, (2010), 1–19.

[12] GIACARDI L., *Federigo Enriques (1871-1946) and the training of mathematics teachers in Italy*, in *Mathematicians in Bologna 1861-1960*, S. Coen (Ed.), Basel, Birkhäuser, (2012), 209–275.

[13] GOODSTEIN J., *L'ascesa e la caduta del mondo di Vito Volterra*, in *La ristrutturazione delle scienze tra le due guerre mondiali*, Vol. I: *L'Europa*, Battimelli, G., De Maria, M. and Rossi, A. (Eds.), La Goliardica, Rome (1984), 289–302.

[14] GUERRAGGIO A. AND NASTASI, P., *Gentile e i matematici italiani. Lettere 1907-1943*, Bollati Boringhieri, Torino 1993.

[15] ISRAEL G. *Le due vie della matematica italiana contemporanea*, in *La ristrutturazione delle scienze tra le due guerre mondiali*. Vol. I: *L'Europa*, Battimelli, G., De Maria, M. and Rossi, A. (Eds.), La Goliardica, Rome (1984), 253–287.

[16] ISRAEL G. *Il Fascismo e la razza. La scienza italiana e le politiche razziali del regime*, Il Mulino, Bologna 2010.

[17] ISRAEL G. AND NURZIA L. *Fundamental trends and conflicts in Italian mathematics between the two World Wars*, Archives internationales d'histoire des sciences, **39**, (1989), 111–143.

[18] LINGUERRI S., *La grande festa della scienza. Eugenio Rignano e Federigo Enriques. Lettere*, Franco Angeli, Milan 2005.

[19] LUCIANO E. AND ROERO C.S., *La Scuola di Giuseppe Peano*, in *Peano e la sua Scuola fra matematica, logica e interlingua*, C. S. Roero (Ed.) Studi e Fonti per la storia dell'Università di Torino, XVII, Deputazione Subalpina di Storia Patria, Torino (2010), I-XVIII, 1-212.

[20] LUCIANO E. AND ROERO C.S., *From Turin to Göttingen: dialogues and correspondence (1879-1923)*, Bollettino di Storia delle Scienze Matematiche, **XXXII**, (2012), 9–232.

[21] MENGHINI M., *La corrispondenza tra Felix Klein e Luigi Cremona (1869-1896)*, in *La corrispondenza di Luigi Cremona (1830-1903)*, vol. II, Quaderni della Rivista di Storia della Scienza, n. 3, (1994), 53–71.

[22] METZGER H., *La Settimana della Scuola di Storia delle Scienze a Roma*, Archeion, **17**, (1935), 203–212.

[23] MIGLIARI R., *L'insegnamento della Geometria Descrittiva e delle sue applicazioni*, in *La Facoltà di Architettura di Roma "La Sapienza" dalle origini al duemila. Discipline, docenti, studenti*, Franchetti Pardo, V. (Ed.), Gangemi Editore, (2001), 279–282

[24] NASTASI P., *Le "Conferenze Americane" di Kelix Klein*, PRISTEM Storia, (2000), 3–4.

[25] NASTASI P., *Considerazioni tumultuarie su Federigo Enriques*, in *Intorno a Enriques*, Scarantino, L.M. (Ed.) Sarzana, Agorà edizioni, (2004), 79–204.

[26] NASTASI P. AND SCIMONE A., *Lettere a Giovanni Vacca*, Quaderni PRISTEM 5, Palermo 1995.

[27] NASTASI T., *Federigo Enriques e la civetta di Atena*. Centro Studi Enriques, Edizioni Plus, Pisa 2010.

[28] NURZIA L., *Relazioni tra le concezioni geometriche di Federigo Enriques e la matematica intuizionistica tedesca*, Physis, **21**, (1979), 157–193

[29] POMPEO FARACOVI O., *Enriques e Severi matematici a confronto nella cultura del Novecento*. Centro Studi Enriques. Agorà Edizioni, with essays of Edoardo Vesentini, Ciro Ciliberto, Aldo Brigaglia, Giorgio Bolondi,Ornella Pompeo Faracovi, Sandra Linguerri, Giorgio Israel, Paolo Bussotti, La Spezia 2004.

[30] POMPEO FARACOVI O., *Enriques, Gentile e la matematica*, in *Da Casati a Gentile. Momenti di storia dell'insegnamento secondario della matematica in Italia*, Giacardi, L. (Ed.), Centro Studi Enriques, 6, Lugano, Lumières Internationales, (2006), 305–321.

[31] ROWE D.E., *Felix Klein's "Erlanger Antrittsrede". A transcription with English Translation and Commentary*, Historia Mathematica, **12**, (1985), 123–141.

[32] SCHUBRING G., *Pure and Applied Mathematics in Divergent Institutional Settings in Germany: the Role and Impact of Felix Klein*, in D. Rowe, J. McCleary eds., *The History of Modern Mathematics*, London, Academic Press, vol. II, (1989), 170– 220

[33] SCHUBRING G., *Felix Klein*, in Furinghetti, F. and Giacardi, L. (Eds.) `http://www.icmihistory.unito.it/portrait/klein.php`, 2008.

[34] SIMILI R., *Per la scienza. Scritti editi e inediti*, Bibliopolis, Napoli 2000.

[35] TRICOMI F., *La mia vita di matematico attraverso la cronistoria dei miei lavori (Bibliografia commentata 1916-1967)*, CEDAM, Padova 1967.

AMS Subject Classification: 01A60, 01A72

Livia GIACARDI,
Department of Mathematics, University of Turin
Via Carlo Alberto 10, 10123 Torino, ITALY
e-mail: livia.giacardi@unito.it

Lavoro pervenuto in redazione il 15.12.2012.