Rend. Sem. Mat. Univ. Pol. Torino
Vol. 67, 3 (2009), 341 – 349

M. Badii

PERIODIC SOLUTIONS FOR A NONLINEAR PARABOLIC EQUATION WITH NONLINEAR BOUNDARY CONDITIONS

Abstract. In this paper we prove the existence of weak periodic solutions for a nonlinear parabolic equations with the Robin periodic boundary condition. The aim will be achieved by reformulating the problem in abstract form and applying some results of the maximal monotone mapping theory joint with the Schauder fixed point theorem.

1. Introduction

Let Ω be a bounded open set of $\mathbb{R}^n$ ($n \geqslant 1$) with smooth boundary $\partial\Omega$ and the outward unit normal vector ν at points of $\partial\Omega$. For given $\omega > 0$, we set $Q := \Omega \times P$ and $\Sigma := \partial\Omega \times P$ where $P := \mathbb{R}/\omega\mathbb{Z}$ denotes the period interval $[0,\omega]$, so the functions defined in Q and Σ are automatically ω-time periodic. We consider the quasilinear parabolic problem in divergence form

$$u_t - \operatorname{div}\mathbf{a}(x,t,u,\nabla u) + f(x,t,u) = h(x,t) \quad \text{in } Q. \tag{1}$$

The formulation of problem is completed specifying the periodic condition

$$u(x,t+\omega) = u(x,t) \quad \text{in } Q,\ \ \omega > 0, \tag{2}$$

and the nonlinear Robin periodic boundary condition

$$-\mathbf{a}(x,t,u,\nabla u)\cdot\nu = \beta(x,t)u + g(x,t,u) \quad \text{on } \Sigma. \tag{3}$$

In this model, the quasilinear operator **a** is assumed to satisfy the standard conditions of Leray–Lions type. The purpose of the paper is to present a result on the existence of at least one weak periodic solution to problem (1)–(3) under quite general assumptions on the nonlinearities.

We will study our problem, making the following structural assumptions on the data

- H_1) $\mathbf{a} : \Omega \times \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ is a Caratheodory function i.e. $\mathbf{a}(\cdot,\cdot,s,\zeta)$ is measurable in (x,t) for any $(s,\zeta) \in \mathbb{R}\times\mathbb{R}^n$ and continuous in (s,ζ) for a.e. $(x,t) \in Q$;

- H_2) there exists a constant $\alpha > 0$ such that

$$(\mathbf{a}(x,t,s,\zeta) - \mathbf{a}(x,t,s,\xi), \zeta - \xi) \geqslant \alpha|\zeta - \xi|^2$$

for all $s \in \mathbb{R}$, ζ, $\xi \in \mathbb{R}^n$ with $\zeta \neq \xi$ and for a.e. $(x,t) \in Q$. Here, $(\cdot,\cdot)$ denotes the inner product in $\mathbb{R}^n$;

H$_3$) $\mathbf{a}(x,t,s,0) = 0$;

H$_4$) there exist a positive function $b \in L^2(Q)$ and a constant $\gamma > 0$ such that

$$|\mathbf{a}(x,t,s,\zeta)| \leqslant b(x,t) + \gamma(|s| + |\zeta|)$$

$\forall s \in \mathbb{R}$, $\zeta \in \mathbb{R}^n$ and a.e. $(x,t) \in Q$;

H$_5$) f is a Caratheodory function such that

$$|f(x,t,\xi)| \leqslant C$$

for a.e. $(x,t) \in Q$;

H$_6$) β is a positive continuous and bounded function such that

$$0 < \beta_1 \leqslant \beta(x,t) \leqslant \beta_2, \ \ \forall (x,t) \in \Sigma;$$

H$_7$) $g : \Sigma \times \mathbb{R} \to \mathbb{R}$ is a Caratheodory function such that for a.e. $(x,t) \in \Sigma$, the mapping $\xi \mapsto g(x,t,\xi)$ is nondecreasing and

$$|g(x,t,\xi)| \leqslant C_1(d(x,t) + |\xi|)$$

$$\xi g(x,t,\xi) \geqslant C_2|\xi|^2 - c_3(x,t)$$

where $d \in L^2(\Sigma)$ and $c_3 \in L^1(\Sigma)$;

H$_8$) $h \in L^2(Q)$.

Problem (1)–(3) can be regarded as a mathematical model for many chemical, physical, biological and ecological phenomena. In order to describe our results and relate them to others in the early literature, we mention that many researchers have studied the existence of periodic solutions for coupled systems, semilinear and quasilinear parabolic equations under either Dirichlet or Neumann boundary conditions using lower and upper solutions (see [1], [5] and references given there). The lower and upper solutions associated with monotone iterations is also used to treat a class of coupled systems of semilinear parabolic equations with nonlinear boundary conditions in [9]. In [4] is studied the existence of at least one periodic solution for a quasilinear parabolic problem in divergence form with Dirichlet boundary data, utilizing the method of lower solution and upper solution. These authors associate to the problem under consideration an auxiliary variational inequality solved by applying the penalty method. Our approach to the periodicity shall be to seek the periodic solutions in an appropriate space of ω-periodic functions rather then look for fixed points for the Poincaré periodic map. Mathematically, it is worth mentioning that the starting point relies on the following Theorem 1 for maximal monotone mappings joint with a suitable fixed point argument.

THEOREM 1 ([2, 3, 7]). *Let L be a linear closed, densely defined operator from the reflexive space V to V^*, L maximal monotone and let A be a bounded hemicontinuous monotone mapping from V to V^*, then $L + A$ is maximal monotone in $V \times V^*$. Moreover, if $L + A$ is coercive, then* $\mathrm{Range}(L + A) = V^*$.

The remaining part of the paper is organized as follows. In Section 2 we choose the functional framework where the periodic solutions are sought and give the notion of weak periodic solution. In Section 3 we use Theorem 1 to prove the existence and uniqueness of periodic solutions for an abstract problem formulated by means of maximal monotone mappings. Finally, Section 4 is devoted to obtaining some crucial estimates and convergences which allow to apply the Schauder fixed point theorem to a nonlinear operator equation and to show the existence of at least one weak periodic solution for problem (1)–(3).

2. Functional framework and definitions

Let us introduce the functional space for the periodic solutions of problem. We consider the Hilbert space

$$V := L^2(P; W^{1,2}(\Omega))$$

endowed with the equivalent norm

$$\|v\|_V := \left(\int_Q |\nabla v(x,t)|^2 dxdt + \int_\Sigma \beta(x,t) |\widetilde{v}(x,t)|^2 d\sigma dt \right)^{1/2}$$

where $\widetilde{v}$ stands for the trace of $v \in V$. It is known that for a regular domain Ω, every $v \in W^{1,2}(\Omega)$ has a trace $\widetilde{v} \in W^{1/2}(\partial\Omega)$ and in view of the trace Sobolev theorem, $W^{1/2}(\partial\Omega) \hookrightarrow L^2(\partial\Omega)$ compactly.

The topological dual space of V is

$$V^* := L^2(P; (W^{1,2}(\Omega))^*)$$

with $\|.\|_*$ norm. The duality pairing between V and V^* will be written as $\langle \cdot, \cdot \rangle$. The notion of weak solution may be introduced as follows.

DEFINITION 1. *A function u is said to be a weak periodic solution of problem* (1)–(3) *if*

$$u \in V \text{ with } u_t \in V^*$$

and

$$\begin{aligned} &\int_Q u_t \eta dxdt + \int_Q (\mathbf{a}(x,t,u,\nabla u), \nabla \eta) dxdt + \int_\Sigma \beta(x,t) \widetilde{u}(x,t) \widetilde{\eta}(x,t) d\sigma dt \\ &\quad + \int_\Sigma g(x,t,\widetilde{u}) \widetilde{\eta}(x,t) d\sigma dt + \int_Q f(x,t,u) \eta(x,t) dxdt \\ &\qquad = \int_Q h(x,t) \eta(x,t) dxdt, \;\; \forall \eta \in V. \end{aligned} \tag{4}$$

Having fixed $w \in L^2(Q)$, we consider the problem

$$\int_Q u_t \eta dxdt + \int_Q (\mathbf{a}(x,t,w,\nabla u), \nabla\eta) dxdt + \int_\Sigma \beta(x,t)\widetilde{u}(x,t)\widetilde{\eta}(x,t) d\sigma dt + \int_\Sigma g(x,t,\widetilde{u})\widetilde{\eta}(x,t) d\sigma dt + \int_Q f(x,t,w)\eta(x,t) dxdt = \int_Q h(x,t)\eta(x,t) dxdt, \ \forall \eta \in V.$$

3. Existence and uniqueness of periodic solutions

In order to use Theorem 1, we must define two mappings L and A. The set

$$\mathcal{D} := \{u \in V : u_t \in V^*\}$$

is dense in V because of the density of $C^\infty(\overline{Q}) \subset \mathcal{D}$ in V. Let

$$L : \mathcal{D} \to V^*$$

be the linear operator given by

$$\langle Lu, \eta \rangle := \int_Q u_t \eta \, dxdt, \ \forall \eta \in V.$$

This operator is closed, skew-adjoint (i.e. $L = -L^*$) and maximal monotone (see [7, Lemma 1.1, p. 313]).

Given $w \in L^2(Q)$ we define

$$A : V \to V^*$$

by setting

$$\langle A(u), \eta \rangle := \int_Q (\mathbf{a}(x,t,w,\nabla u), \nabla\eta) dxdt + \int_\Sigma \beta(x,t)\widetilde{u}(x,t)\widetilde{\eta}(x,t) d\sigma dt + \int_\Sigma g(x,t,\widetilde{u})\widetilde{\eta}(x,t) d\sigma dt, \ \forall \eta \in V.$$

The next result summarizes the properties of the operator A.

PROPOSITION 1. *If the assumptions H_1)–H_4) and H_6), H_7) are fulfilled, then the mapping A is:*

i) hemicontinuous;

ii) monotone;

iii) coercive.

Proof. i) The hemicontinuity follows from the Hölder inequality. In fact,

$$|\langle A(u),\eta\rangle| \leqslant \left[\|b\|_{L^2(Q)}+\gamma\|w\|_{L^2(Q)}+(\gamma+1+\frac{C_1}{\beta_1})\|u\|_V+\frac{C_1}{\sqrt{\beta_1}}\|d\|_{L^2(\Sigma))}\right]\|\eta\|_V$$

by which

$$\|A(u)\|_* \leqslant \left[\|b\|_{L^2(Q)}+\gamma\|w\|_{L^2(Q)}+(\gamma+1+\frac{C_1}{\beta_1})\|u\|_V+\frac{C_1}{\sqrt{\beta_1}}\|d\|_{L^2(\Sigma))}\right].$$

The proof of i) is completed by applying [6, Theorems 2.1. and 2.3].
ii) The monotonicity is a consequence of

$$\begin{aligned}\langle A(u)-A(v),u-v\rangle = &\int_Q(\mathbf{a}(x,t,w,\nabla u)-\mathbf{a}(x,t,w,\nabla v),\nabla(u-v))dxdt\\ &+\int_\Sigma \beta(x,t)|\widetilde{u}(x,t)-\widetilde{v}(x,t)|^2 d\sigma dt\\ &+\int_\Sigma (g(x,t,\widetilde{u})-g(x,t,\widetilde{v}))(\widetilde{u}(x,t)-\widetilde{v}(x,t))d\sigma dt \;\geqslant\; 0.\end{aligned}$$

iii) Coercivity. One has

$$\begin{aligned}\langle A(u),u\rangle \;=\;& \textstyle\int_Q(\mathbf{a}(x,t,w,\nabla u),\nabla u)dxdt+\int_\Sigma\beta(x,t)|\widetilde{u}(x,t)|^2d\sigma dt\\ &+\textstyle\int_\Sigma g(x,t,\widetilde{u})\widetilde{u}(x,t)d\sigma dt\\ \geqslant\;& \alpha\textstyle\int_Q|\nabla u(x,t)|^2dxdt+\int_\Sigma\beta(x,t)|\widetilde{u}(x,t)|^2d\sigma dt\\ &+\tfrac{C_2}{\beta_2}\textstyle\int_\Sigma\beta(x,t)|\widetilde{u}(x,t)|^2d\sigma dt-\int_\Sigma c_3(x,t)d\sigma dt\\ \geqslant\;& \min(\alpha,1+\tfrac{C_2}{\beta_2})\|u\|_V^2-\textstyle\int_\Sigma c_3(x,t)d\sigma dt.\end{aligned}$$

Thus,

$$\frac{\langle A(u),u\rangle}{\|u\|_V} \geqslant \min(\alpha,1+\frac{C_2}{\beta_2})\|u\|_V-\frac{\int_\Sigma c_3(x,t)d\sigma dt}{\|u\|_V} \;\to\; +\infty \;\text{ as }\; \|u\|_V\to+\infty.$$

□

Besides, let $G\in V^*$ be the linear functional defined as

$$\langle G,\eta\rangle := -\int_Q f(x,t,w)\eta(x,t)dxdt+\int_Q h(x,t)\eta(x,t)dxdt,\;\;\forall\eta\in V$$

then, problem (4) can be reformulated in the following abstract form:

(5) $$Lu+A(u)=G.$$

Now, we state the main result of the section.

PROPOSITION 2. *Let $w \in L^2(Q)$ be given and assuming H_1)–H_7), the problem* (5) *has a unique weak periodic solution.*

Proof. The existence of weak periodic solutions descends from Theorem 1. Uniqueness is a consequence of the strict monotonicity. ☐

4. Fixed points

The existence of weak periodic solutions to (1)–(3) will be based on the research of fixed points for the nonlinear mapping

$$\Phi : L^2(Q) \to L^2(Q)$$

defined by

$$\Phi(w) = u$$

where u is the unique weak periodic solution of (3.1) corresponding to $w \in L^2(Q)$. The mapping Φ is well-defined. To show its continuity we will prove some very important estimates and convergences needed to apply the Schauder fixed point theorem.

Let $w_n \in L^2(Q)$ be a sequence convergent strongly to w in $L^2(Q)$. Moreover, let u_n denote the weak periodic solution of the problem

$$\begin{aligned}(6) \quad & \int_Q u_{nt}(x,t)\eta(x,t)dxdt + \int_Q (\mathbf{a}(x,t,w_n,\nabla u_n),\nabla\eta)dxdt \\ & + \int_\Sigma \beta(x,t)\widetilde{u}_n(x,t)\widetilde{\eta}(x,t)d\sigma dt + \int_\Sigma g(x,t,\widetilde{u}_n)\widetilde{\eta}(x,t)d\sigma dt \\ & + \int_Q f(x,t,w_n)\eta(x,t)dxdt = \int_Q h(x,t)\eta(x,t)dxdt.\end{aligned}$$

Setting $\eta = u_n$ as a test function in (6), we have

$$\begin{aligned}(7) \quad & \int_Q u_{nt}(x,t)u_n(x,t)dxdt + \int_Q (\mathbf{a}(x,t,w_n,\nabla u_n),\nabla u_n)dxdt \\ & + \int_\Sigma \beta(x,t)|\widetilde{u}_n(x,t)|^2 d\sigma dt + \int_\Sigma g(x,t,\widetilde{u}_n)\widetilde{u}_n(x,t)d\sigma dt \\ & + \int_Q f(x,t,w_n)u_n(x,t)dxdt = \int_Q h(x,t)u_n(x,t)dxdt.\end{aligned}$$

Conditions H_2), H_3), H_7), the periodicity and the Young inequality give us

$$\begin{aligned}& \alpha\int_Q |\nabla u_n(x,t)|^2 dxdt + \int_\Sigma \beta(x,t)|\widetilde{u}_n(x,t)|^2 d\sigma dt \\ & \quad + \frac{C_2}{\beta_2}\int_\Sigma \beta(x,t)|\widetilde{u}_n(x,t)|^2 d\sigma dt - \int_\Sigma c_3(x,t)d\sigma dt \\ & \leqslant \frac{1}{2\varepsilon}\int_Q |f(x,t,w_n)|^2 dxdt + \varepsilon\int_Q |u_n(x,t)|^2 dxdt + \frac{1}{2\varepsilon}\int_Q |h(x,t)|^2 dxdt\end{aligned}$$

and taking into account the equivalence of the norms in V, we have

$$\left(\min(\alpha, 1+\frac{C_2}{\beta_2}) - H\varepsilon\right)\left(\int_Q |\nabla u_n(x,t)|^2 dxdt + \int_\Sigma \beta(x,t)|\widetilde{u}_n(x,t)|^2 d\sigma dt\right)$$
$$\leqslant \frac{1}{2\varepsilon}\int_Q |f(x,t,w_n)|^2 dxdt + \int_\Sigma |c_3(x,t)| d\sigma dt + \frac{1}{2\varepsilon}\int_Q |h(x,t)|^2 dxdt \leqslant C'.$$

This permits us to obtain the classical energy estimate

$$(8) \qquad \int_Q |\nabla u_n(x,t)|^2 dxdt + \int_\Sigma \beta(x,t)|\widetilde{u}_n(x,t)|^2 d\sigma dt \leqslant C''$$

where the positive constant C'' is independent of n. From (6) and the energy estimate (8) we get that u_{nt} is bounded in the V^* norm. This provided the boundedness of u_n in the norm of the set $\mathcal{D}$ i.e.

$$\|u_n\|_{\mathcal{D}} \leqslant L, \ \forall n \in N.$$

Thus, we can select a subsequence, still denoted by u_n such that

$$u_n \rightharpoonup u \ \text{ in } \mathcal{D} \ \text{ as } n \to +\infty.$$

By a result in [5, Theorem 5.1], the sequence u_n is precompact in $L^2(Q)$ therefore,

$$u_n \to u \ \text{ in } L^2(Q) \ \text{ and a.e. in } Q.$$

Furthermore, according to the trace theorem (see [8, Theorem 3.4.1]) one has

$$\widetilde{u}_n \to \widetilde{u} \ \text{ in } L^2(P; L^2(\partial\Omega)).$$

LEMMA 1. *The sequence ∇u_n converges strongly to ∇u in $L^2(P;(L^2(\Omega))^n)$.*

Proof. By (8), H_4) and the strong convergence of w_n to w in $L^2(Q)$, we have that $\mathbf{a}(x,t,w_n,\nabla u_n)$ is bounded in $L^2(Q)$, that is

$$\mathbf{a}(x,t,w_n,\nabla u_n) \rightharpoonup \mu \text{ in } L^2(P;(L^2(\Omega))^n).$$

Letting $n \to +\infty$ in (7) we obtain

$$\lim_n \int_Q (\mathbf{a}(x,t,w_n,\nabla u_n), \nabla u_n) dxdt = -\int_\Sigma \beta(x,t)|\widetilde{u}(x,t)|^2 d\sigma dt$$
$$-\int_\Sigma g(x,t,\widetilde{u})\widetilde{u}(x,t) d\sigma dt - \int_Q f(x,t,w)u(x,t) dxdt + \int_Q h(x,t)u(x,t) dxdt.$$

Moreover,

$$\int_Q u_{nt}(x,t)u(x,t) dxdt + \int_Q (\mathbf{a}(x,t,w_n,\nabla u_n), \nabla u) dxdt$$
$$+\int_\Sigma \beta(x,t)\widetilde{u}_n(x,t)\widetilde{u}(x,t) d\sigma dt + \int_\Sigma g(x,t,\widetilde{u}_n)\widetilde{u}(x,t) d\sigma dt$$
$$+\int_Q f(x,t,w_n)u(x,t) dxdt = \int_Q h(x,t)u(x,t) dxdt.$$

Taking the limit on $n \to +\infty$, we infer that

$$\lim_n \int_Q (\mathbf{a}(x,t,w_n,\nabla u_n),\nabla u)dxdt = -\int_\Sigma \beta(x,t)|\widetilde{u}(x,t)|^2 d\sigma dt$$
$$-\int_\Sigma g(x,t,\widetilde{u})\widetilde{u}(x,t)d\sigma dt - \int_Q f(x,t,w)u(x,t)dxdt + \int_Q h(x,t)u(x,t)dxdt.$$

Since

$$\begin{aligned}
&\alpha\int_Q |\nabla(u_n(x,t)-u(x,t))|^2 dxdt\\
&\leqslant \int_Q (\mathbf{a}(x,t,w_n,\nabla u_n) - (\mathbf{a}(x,t,w_n,\nabla u),\nabla(u_n(x,t)-u(x,t))dxdt\\
&= \int_Q (\mathbf{a}(x,t,w_n,\nabla u_n),\nabla u_n)dxdt - \int_Q (\mathbf{a}(x,t,w_n,\nabla u_n),\nabla u)dxdt\\
&-\int_Q (\mathbf{a}(x,t,w_n,\nabla u),\nabla(u_n(x,t)-u(x,t)))dxdt,
\end{aligned}$$

passing to the limit in n given the assumption $\mathrm{H}_1)$ and $\nabla u_n \rightharpoonup \nabla u$ in $L^2(P;(L^2(\Omega))^n)$, one proves that

$$\lim_n \int_Q |\nabla(u_n(x,t)-u(x,t))|^2 dxdt = 0$$

i.e.

$$\nabla u_n \to \nabla u \text{ in } L^2(P;(L^2(\Omega))^n) \text{ and a.e. in } Q.$$

Hence,

$$\mathbf{a}(x,t,w,\nabla u) = \mu \text{ a.e. in } Q$$

so that

$$\mathbf{a}(x,t,w_n,\nabla u_n) \rightharpoonup \mathbf{a}(x,t,w,\nabla u) \text{ in } L^2(P;(L^2(\Omega))^n).$$

□

LEMMA 2. *The mapping Φ is continuous.*

Proof. The above convergences

$$\begin{aligned}
&w_n \to w \text{ in } L^2(Q)\\
&u_n \to u \text{ in } L^2(Q) \text{ and a.e. in } Q\\
&\nabla u_n \to \nabla u \text{ in } L^2(P;(L^2(\Omega))^n) \text{ and a.e. in } Q\\
&\widetilde{u}_n \to \widetilde{u} \text{ in } L^2(P;L^2(\partial\Omega))\\
&\mathbf{a}(x,t,w_n,\nabla u_n) \rightharpoonup \mathbf{a}(x,t,w,\nabla u) \text{ in } L^2(P;(L^2(\Omega))^n)
\end{aligned}$$

enable us to conclude that $\Phi(w_n) = u_n$ converges stongly to $\Phi(w) = u$ in $L^2(Q)$. □

LEMMA 3. *There exists a constant $R > 0$ such that*

$$\|\Phi(w)\|_{L^2(Q)} \leqslant R, \ \forall w \in L^2(Q).$$

Proof. Letting $n \to +\infty$ in (8), the assertion of the lemma is proved. □

Since $\Phi(L^2(Q) \subset L^2(Q)$ and the embedding $\mathcal{D} \hookrightarrow L^2(Q)$ is compact, Φ is a compact operator from $L^2(Q)$ to itself.

Finally, we state the main result of the paper.

THEOREM 2. *Under the assumptions $H_1)$–$H_7)$, there exists at least one fixed point of Φ.*

Proof. Lemmas 2 and 3 imply that the mapping Φ is both continuous and compact, then by the Schauder fixed point theorem there exists at least one fixed point for the mapping Φ which is a weak periodic solution to (1)–(3). □

References

[1] AMMAN H., *Periodic solutions of semilinear parabolic equations*, Nonlinear Analysis Academic Press, New York 1978, 1–29.

[2] BARBU V., *Nonlinear semigrups and differential equations in Banach spaces*, Noordhoff International Publishing, Leiden 1976.

[3] BROWDER F.E., *Nonlinear maximal monotone operators in Banach space*, Math. Ann. **175** (1968), 89–113.

[4] DEUEL J. AND HESSE P., *Nonlinear parabolic boundary value problems with upper and lower solutions*, Israel Journal of Mathematics **29** (1978), 92–104.

[5] KOLESOV JU.S., *Periodic solutions of quasilinear parabolic equations of second order*, Trans. Moscow Math. Soc. **21** (1970), 114–146.

[6] KRASNOSELSKII M.A., *Topological methods in the theory of nonlinear integral equations*, Pergamon Press, New York 1964.

[7] LIONS J.L., *Quelques méthodes de résolution des problèmes aux limites non linéaires*, Dunod, Paris 1969.

[8] MORREY C.B. JR, *Multiple integrals in the calculus of variation*, Springer-Verlag, New York 1966.

[9] PAO C.V., *Periodic solutions of parabolic with nonlinear boundary conditions*, J. Math. Anal. Appl. **234** (1999), 695–716.

AMS Subject Classification: 35B10, 35D05, 35K60, 47J35

Maurizio BADII,
Dipartimento di Matematica “G. Castelnuovo”, Università di Roma “La Sapienza”,
P.le Aldo Moro 5, 00185 Roma, ITALIA
e-mail: `badii@mat.uniroma1.it`

Lavoro pervenuto in redazione il 02.09.2008