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INEXTENSIBLE NETWORKS WITH BENDING AND TWISTING EFFECTS

Abstract. Families of inextensible fibers forming a surface are considered. Each fiber supports a twisting couple proportional to the torsion of the fiber. The strain energy density is written in an additive form, such that the contributions due to shearing, twisting and bending effects are taken into account separately. The equilibrium equations, here obtained, are a particular case of the ones obtained by Luo and Steigmann in [1].

1. Introduction

We are interested in the theory of inextensible networks, in particular in the case in which a set of inextensible fibers forms a surface with bending stiffness and in which the twisting fiber effects are taken into account, such that we can model the static behaviour of textile fabrics.

In 1986, Wang and Pipkin [5] formulated a theory of inextensible nets with bending stiffness. The resulting continuum theory is a special form of finite-deformation plate theory in which each fiber has a bending couple proportional to its curvature.

In 2001, a theory of bending and twisting effects in three-dimensional deformation of an inextensible network is presented by Luo and Steigmann [1]. They derive the Euler-Lagrange equations and boundary conditions by using the minimum-energy principle. (A simplified version of these equations represents the equilibrium equations obtained by Wang and Pipkin [5].)

The aim of this work consists of finding the equilibrium equations for a net of inextensible fibers taking into account the twist and the bending of fibers. In section 2, we give the constitutive hypotheses. In section 3, we obtain a set of equations where the effect of the twist of the fibers on the deformation of the sheet is exibited. We assume that each fiber in the fabric supports a twisting couple, since we are looking at some expressions which take into account the twist of fibers we assume that each couple is proportional to the torsion of the fiber. In section 4, we focus on the energy of strain for sets of fibers that undergo shear and twist deformations.

2. Inextensible fibers, constitutive hypotheses

We consider two families of inextensible fibers forming a surface that initially lies in a region B of the (x,y)-plane. We assume that initially the first family of fibers, d_1, stays parallel to the x axis and that the second family of fibers, d_2, stays parallel to the y

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axis. We suppose that fibers are continuously distributed so that every line x=constant or y=constant in B is regarded as a fiber. Each fiber is permanently identified by its initial coordinate, x or y. We suppose that cross-sections of each fiber remain plane, suffer no strain, and are normal to the fiber in every configuration (Bernoulli-Euler hypotheses). We denote the position in the current configuration with $\boldsymbol{r}$(x,y), namely the point of the fibers that initially lies in the position (x,y) moves to the place $\boldsymbol{r}$(x,y) in three-dimensional space.

Let

$$\boldsymbol{d}_1 = \frac{\partial \boldsymbol{r}}{\partial x} = \boldsymbol{r}_{,x} \qquad \boldsymbol{d}_2 = \frac{\partial \boldsymbol{r}}{\partial y} = \boldsymbol{r}_{,y}$$

be the tangential vectors to the curve occupied by a fiber y=constant and x=constant, respectively, when the sheet is deformed. We postulate that no part of any fiber can change its length in any admissible deformation so the vectors $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$ are unit tangent vectors [3]. Since x and y are the arc length of the d_1 and d_2 lines, Frenet's formulas allow us to attach to each fiber the normal vector $\boldsymbol{n}$ and the binormal vector $\boldsymbol{b}$, so for the fiber d_1 the triad $\{\boldsymbol{d}_1, \boldsymbol{n}_1, \boldsymbol{b}_1\}$ satisfies:

$$\begin{cases} \dfrac{\partial \boldsymbol{d}_1}{\partial x} = k_1 \boldsymbol{n}_1 \\ \dfrac{\partial \boldsymbol{n}_1}{\partial x} = -k_1 \boldsymbol{d}_1 + \tau_1 \boldsymbol{b}_1 \\ \dfrac{\partial \boldsymbol{b}_1}{\partial x} = -\tau_1 \boldsymbol{n}_1 \end{cases} \tag{1}$$

with k_1 the curvature of the d_1 line and τ_1 torsion of the d_1 line. Similarly, for the fiber d_2 we introduce the Frenet triad $\{\boldsymbol{d}_2, \boldsymbol{n}_2, \boldsymbol{b}_2\}$.

The sets of fibers d_1 and d_2 are related through the angle of shear γ, that is defined by the relation

$$\sin\gamma = \boldsymbol{d}_1 \cdot \boldsymbol{d}_2$$

this angle describes the local distortion of the sheet. Moreover, we introduce the normal vector:

$$\boldsymbol{N} = \frac{\boldsymbol{d}_1 \times \boldsymbol{d}_2}{|\boldsymbol{d}_1 \times \boldsymbol{d}_2|}.$$

3. Twisting effects

Wang and Pipkin in [5] assume that each fiber in the fabric supports a bending couple proportional to the curvature of the fiber:

$$\boldsymbol{c}_1 = \Gamma \boldsymbol{d}_1 \times \boldsymbol{d}_{1,x} \qquad \boldsymbol{c}_2 = \Gamma \boldsymbol{d}_2 \times \boldsymbol{d}_{2,y}\,.$$

The stiffness coefficient Γ is the same positive constant for all the fibers. They find the following equations:

$$\begin{cases} \boldsymbol{t}_1 = T_1 \boldsymbol{d}_1 + S \boldsymbol{d}_2 - \Gamma \boldsymbol{d}_{1,xx} \\ \boldsymbol{t}_2 = T_2 \boldsymbol{d}_2 + S \boldsymbol{d}_1 - \Gamma \boldsymbol{d}_{2,yy} \\ \boldsymbol{t}_{1,x} + \boldsymbol{t}_{2,y} + \boldsymbol{f} = \boldsymbol{0} \end{cases} \tag{2}$$

with: T_1 and T_2 fiber tensions (reactions to the constraints of fiber inextensibility), S the shearing stress, $\boldsymbol{t}_1$ the force per unit length exerted across a d_2-line $x=x_0$ by the material on the side $x>x_0$ on the material on the other side ($x\leq x_0$), $\boldsymbol{t}_2$ the force per unit length exerted across a d_1-line $y=y_0$ by the material on the side $y>y_0$ on the material on the other side ($y\leq y_0$).
The equations (2) include the effects of couple-stress vectors that account for bending couples in the deformed sheet.

The aim of our work is to find the equations that express the effect of the twist of the fibers on the deformation of the sheet. We assume that each fiber in the fabric supports a twisting couple, since we are looking at expressions which take into account the twist of fibers we assume that each couple is proportional to the torsion, τ_1 or τ_2, of the fiber. Secondly, chosen a set of fibers, say d_1-lines, we want that the vector associated to the couple is directed like the tangent vector $\boldsymbol{d}_1$, consequently we choose the twisting couples:

$$\boldsymbol{i}_1 = \Lambda \boldsymbol{b}_1 \times \boldsymbol{b}_{1,x} \qquad \boldsymbol{i}_2 = \Lambda \boldsymbol{b}_2 \times \boldsymbol{b}_{2,y} \tag{3}$$

with Λ twisting coefficient. Recalling $(1)_3$ we have: $\boldsymbol{i}_1 = \Lambda\tau_1\boldsymbol{d}_1$ and $\boldsymbol{i}_2 = \Lambda\tau_2\boldsymbol{d}_2$.

Taken a directed arc $d\boldsymbol{r} = \boldsymbol{d}_1 dx + \boldsymbol{d}_2 dy$, whose initial length is ds, the force $\boldsymbol{t}ds$ exerted across it is:

$$\boldsymbol{t}ds = \boldsymbol{t}_1 dy - \boldsymbol{t}_2 dx$$

with $\boldsymbol{t}_1$ and $\boldsymbol{t}_2$ the forces defined before. The couple $\boldsymbol{i}$ per unit initial length across a directed arc is given by:

$$\boldsymbol{i}ds = \boldsymbol{i}_1 dy - \boldsymbol{i}_2 dx.$$

For translational equilibrium we have that, for any part of the sheet, the sum of the external forces and of the forces exerted through the boundary lines is null, so the following equation holds:

$$\oint (\boldsymbol{t}_1 dy - \boldsymbol{t}_2 dx) + \int\int \boldsymbol{f} dx dy = \boldsymbol{0} \tag{4}$$

where the area integral is taken over the considered part, the line integral is taken around its perimeter and $\boldsymbol{f}$ is an externally imposed force per unit of initial area that acts on the surface of the sheet.
Equation (4) holds for any part of the sheet, consequentely, if $\boldsymbol{t}_1$ and $\boldsymbol{t}_2$ are smooth, using the divergence theorem we obtain:

$$\boldsymbol{t}_{1,x} + \boldsymbol{t}_{2,y} + \boldsymbol{f} = \boldsymbol{0}. \tag{5}$$

For rotational equilibrium, we have that the moment of the forces exerted on any part of the sheet plus the twisting couples has to be zero:

$$\oint \boldsymbol{r} \times (\boldsymbol{t}_1 dy - \boldsymbol{t}_2 dx) + \int\int \boldsymbol{r} \times \boldsymbol{f} dx dy + \oint (\boldsymbol{i}_1 dy - \boldsymbol{i}_2 dx) = \boldsymbol{0}. \tag{6}$$

The associated differential form is:

$$(\boldsymbol{r} \times \boldsymbol{t}_1 + \boldsymbol{i}_1),_x + (\boldsymbol{r} \times \boldsymbol{t}_2 + \boldsymbol{i}_2),_y + \boldsymbol{r} \times \boldsymbol{f} = \boldsymbol{0}. \tag{7}$$

If (5) is satisfied, then (7) becomes:

$$\boldsymbol{r},_x \times \boldsymbol{t}_1 + \boldsymbol{r},_y \times \boldsymbol{t}_2 + \boldsymbol{i}_{1,x} + \boldsymbol{i}_{2,y} = \boldsymbol{0}. \tag{8}$$

Recalling that $\boldsymbol{d}_1 = \boldsymbol{r},_x$ $\boldsymbol{d}_2 = \boldsymbol{r},_y$, using (1) and (3), we find:

$$\boldsymbol{d}_1 \times \boldsymbol{t}_1 + \Lambda\tau_1 k_1(\boldsymbol{b}_1 \times \boldsymbol{d}_1) + \Lambda(\tau_1),_x \boldsymbol{d}_1 + \boldsymbol{d}_2 \times \boldsymbol{t}_2 + \Lambda\tau_2 k_2(\boldsymbol{b}_2 \times \boldsymbol{d}_2) + \Lambda(\tau_2),_y \boldsymbol{d}_2 = \boldsymbol{0}. \tag{9}$$

Starting from equation (9), we obtain:

$$\boldsymbol{d}_1 \times [\boldsymbol{t}_1 - \Lambda\tau_1 k_1 \boldsymbol{b}_1] + \Lambda(\tau_1),_x \boldsymbol{d}_1 = -\boldsymbol{d}_2 \times [\boldsymbol{t}_2 - \Lambda\tau_2 k_2 \boldsymbol{b}_2] - \Lambda(\tau_2),_y \boldsymbol{d}_2.$$

If the torsion remains constant along the fibers, for example in the case of helicoidal fibers, $(\tau_1),_x$ and $(\tau_2),_y$ vanish. The equation above reduces to

$$\boldsymbol{d}_1 \times [\boldsymbol{t}_1 - \Lambda\tau_1 k_1 \boldsymbol{b}_1] = [\boldsymbol{t}_2 - \Lambda\tau_2 k_2 \boldsymbol{b}_2] \times \boldsymbol{d}_2$$

where the first member of the equation is orthogonal to $\boldsymbol{d}_1$ and the second member is orthogonal to $\boldsymbol{d}_2$, and since the two member are equal one to the other they have a common value say $D\boldsymbol{d}_1 \times \boldsymbol{d}_2$. Consequentely

$$\boldsymbol{d}_1 \times [\boldsymbol{t}_1 - \Lambda\tau_1 k_1 \boldsymbol{b}_1] = [\boldsymbol{t}_2 - \Lambda\tau_2 k_2 \boldsymbol{b}_2] \times \boldsymbol{d}_2 = D\boldsymbol{d}_1 \times \boldsymbol{d}_2$$

and

$$\boldsymbol{d}_1 \times [\boldsymbol{t}_1 - \Lambda\tau_1 k_1 \boldsymbol{b}_1 - D\boldsymbol{d}_2] = [\boldsymbol{t}_2 - \Lambda\tau_2 k_2 \boldsymbol{b}_2 - D\boldsymbol{d}_1] \times \boldsymbol{d}_2 = \boldsymbol{0}.$$

Hence, the vector $[\boldsymbol{t}_1 - \Lambda\tau_1 k_1 \boldsymbol{b}_1 - D\boldsymbol{d}_2]$ is parallel to $\boldsymbol{d}_1$, say it has a value $V_1\boldsymbol{d}_1$. Similarly, $[\boldsymbol{t}_2 - \Lambda\tau_2 k_2 \boldsymbol{b}_2 - D\boldsymbol{d}_1]$ is parallel to $\boldsymbol{d}_2$, say it has a value $V_2\boldsymbol{d}_2$. Consequently, we find:

$$\begin{cases} \boldsymbol{t}_1 = D\boldsymbol{d}_2 + \Lambda\tau_1 k_1 \boldsymbol{b}_1 + V_1\boldsymbol{d}_1 \\ \boldsymbol{t}_2 = D\boldsymbol{d}_1 + \Lambda\tau_2 k_2 \boldsymbol{b}_2 + V_2\boldsymbol{d}_2 \\ \dfrac{\partial}{\partial x}(\tau_1) = 0 \\ \dfrac{\partial}{\partial y}(\tau_2) = 0 \end{cases} \tag{10}$$

4. Strain energy

In the work of Wang and Pipkin [5] the energy of strain W has an additive form:

$$W = W_0(\boldsymbol{d}_1 \cdot \boldsymbol{d}_2) + \frac{1}{2}\Gamma(\boldsymbol{d}_1,_x \cdot \boldsymbol{d}_1,_x + \boldsymbol{d}_2,_y \cdot \boldsymbol{d}_2,_y). \tag{11}$$

The energy component W_0 is due to the shearing stress, since this stress component is that which resists to the changes in the angle between the fibers d_1 and d_2, they assume that W_0 is a function of $\boldsymbol{d}_1 \cdot \boldsymbol{d}_2$; the second component of the strain energy is associated to bending, it is a quadratic form in the fiber curvatures. Equation (11) can be written in the following explicit form:

$$W = W_0(\sin\gamma) + \frac{1}{2}\Gamma[(k_1)^2 + (k_2)^2]. \tag{12}$$

In [1], Luo and Steigmann assume the strain energy to be a function of shear, of the curvatures k_1, k_2 of the fibers (they denote them by η) and of the twists β_1, β_2 of the fibers. Applying the minimum-energy principle they derive the Euler-Lagrange equations ([1](5.5)) in the form:

$$\begin{cases} \boldsymbol{F}_1 = -\dfrac{\partial}{\partial x}\left(\dfrac{\partial W}{\partial k_1}\boldsymbol{n}_1\right) + \dfrac{\partial W}{\partial \sin\gamma}\boldsymbol{d}_2 + \dfrac{\partial W}{\partial \beta_1}k_1\boldsymbol{b}_1 + T_1\boldsymbol{d}_1 \\ \boldsymbol{F}_2 = -\dfrac{\partial}{\partial y}\left(\dfrac{\partial W}{\partial k_2}\boldsymbol{n}_2\right) + \dfrac{\partial W}{\partial \sin\gamma}\boldsymbol{d}_1 + \dfrac{\partial W}{\partial \beta_2}k_2\boldsymbol{b}_2 + T_2\boldsymbol{d}_2 \\ \dfrac{\partial}{\partial x}\left(\dfrac{\partial W}{\partial \beta_1}\right) = 0 \\ \dfrac{\partial}{\partial y}\left(\dfrac{\partial W}{\partial \beta_2}\right) = 0 \\ \dfrac{\partial \boldsymbol{F}_1}{\partial x} + \dfrac{\partial \boldsymbol{F}_2}{\partial y} + \boldsymbol{f} = \boldsymbol{0} \end{cases} \tag{13}$$

with $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ the respective forces on cross sections of $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$ lines. Using as special case (12) in (13), they obtain the equations:

$$\begin{cases} \boldsymbol{F}_1 = T_1\boldsymbol{d}_1 + \dfrac{\mathrm{d}\ W_0}{\mathrm{d}\ \sin\gamma}\boldsymbol{d}_2 - \Gamma\boldsymbol{d}_{1,xx} \\ \boldsymbol{F}_2 = T_2\boldsymbol{d}_2 + \dfrac{\mathrm{d}\ W_0}{\mathrm{d}\ \sin\gamma}\boldsymbol{d}_1 - \Gamma\boldsymbol{d}_{2,yy} \\ \dfrac{\partial \boldsymbol{F}_1}{\partial x} + \dfrac{\partial \boldsymbol{F}_2}{\partial y} + \boldsymbol{f} = \boldsymbol{0} \end{cases} \tag{14}$$

that corresponds to the equations (2) and (5) found by Wang and Pipkin.

We are looking at sets of fibers that undergo twist and shear so it is meaningful to consider the deformation energy W per unit of initial area in the form:

$$W = W_0(\boldsymbol{d}_1 \cdot \boldsymbol{d}_2) + \frac{1}{2}\Lambda(\boldsymbol{b}_{1,x} \cdot \boldsymbol{b}_{1,x} + \boldsymbol{b}_{2,y} \cdot \boldsymbol{b}_{2,y})$$

or equivalently, using Frenet formulas and expressing W_0 through the angle of shear:

$$W = W_0(\sin\gamma) + \frac{1}{2}\Lambda[(\tau_1)^2 + (\tau_2)^2]. \tag{15}$$

We recall formulas (2.12) find by Luo [1], which relate the twists β_1, β_2 of the fibers with the torsions τ_1, τ_2:

$$\beta_1 = \tau_1 + \frac{\partial \theta_1}{\partial x} \qquad \beta_2 = \tau_2 + \frac{\partial \theta_2}{\partial y}. \tag{16}$$

in (16), θ_1 is the angle defined by:

$$\begin{cases} \boldsymbol{a}_2 = \cos\theta_1 \boldsymbol{n}_1 + \sin\theta_1 \boldsymbol{b}_1 \\ \boldsymbol{a}_3 = -\sin\theta_1 \boldsymbol{n}_1 + \cos\theta_1 \boldsymbol{b}_1 \end{cases} \tag{17}$$

where $\{\boldsymbol{d}_1, \boldsymbol{a}_2, \boldsymbol{a}_3\}$ is an orthonormal basis. The angle θ_2 is defined in a similar way.

Differentiating with respect to β_1 and β_2, respectively, equations (16) we have:

$$\frac{\partial \tau_1}{\partial \beta_1} = 1 \qquad \frac{\partial \tau_2}{\partial \beta_2} = 1.$$

By mean of (15), equations (13) read:

$$\begin{cases} \boldsymbol{F}_1 = T_1 \boldsymbol{d}_1 + \dfrac{\mathrm{d}\ W_0}{\mathrm{d}\ \sin\gamma} \boldsymbol{d}_2 + \dfrac{\partial W}{\partial \tau_1} \dfrac{\partial \tau_1}{\partial \beta_1} k_1 \boldsymbol{b}_1 \\ \boldsymbol{F}_2 = T_2 \boldsymbol{d}_2 + \dfrac{\mathrm{d}\ W_0}{\mathrm{d}\ \sin\gamma} \boldsymbol{d}_1 + \dfrac{\partial W}{\partial \tau_2} \dfrac{\partial \tau_2}{\partial \beta_2} k_2 \boldsymbol{b}_2 \\ \dfrac{\partial}{\partial x}\left(\dfrac{\partial W}{\partial \beta_2}\right) = \dfrac{\partial}{\partial x}\left(\dfrac{1}{2}\Lambda 2\tau_1\right) = \Lambda \dfrac{\partial}{\partial x}(\tau_1) = 0 \\ \dfrac{\partial}{\partial y}\left(\dfrac{\partial W}{\partial \beta_2}\right) = \dfrac{\partial}{\partial y}\left(\dfrac{1}{2}\Lambda 2\tau_2\right) = \Lambda \dfrac{\partial}{\partial y}(\tau_2) = 0 \\ \dfrac{\partial \boldsymbol{F}_1}{\partial x} + \dfrac{\partial \boldsymbol{F}_2}{\partial y} + \boldsymbol{f} = \boldsymbol{0} \end{cases} \tag{18}$$

since the torsions τ_1 and τ_2 are constant along the respective fibers, we get the final form:

$$\begin{cases} \boldsymbol{F}_1 = T_1 \boldsymbol{d}_1 + \dfrac{\mathrm{d}\ W_0}{\mathrm{d}\ \sin\gamma} \boldsymbol{d}_2 + \Lambda \tau_1 k_1 \boldsymbol{b}_1 \\ \boldsymbol{F}_2 = T_2 \boldsymbol{d}_2 + \dfrac{\mathrm{d}\ W_0}{\mathrm{d}\ \sin\gamma} \boldsymbol{d}_1 + \Lambda \tau_2 k_2 \boldsymbol{b}_2 \\ \Lambda \dfrac{\partial}{\partial x}(\tau_1) = 0 \\ \Lambda \dfrac{\partial}{\partial y}(\tau_2) = 0 \\ \dfrac{\partial \boldsymbol{F}_1}{\partial x} + \dfrac{\partial \boldsymbol{F}_2}{\partial y} + \boldsymbol{f} = \boldsymbol{0} \end{cases} \tag{19}$$

that could be easily compared with equations (5) and (10).

If also the bending effects are considered, the strain-energy function may be written as:

$$W = W_0(\sin\gamma) + \frac{1}{2}\Lambda[(\tau_1)^2 + (\tau_2)^2] + \frac{1}{2}\Gamma[(k_1)^2 + (k_2)^2] \tag{20}$$

the following equations hold:

$$\begin{cases} \boldsymbol{F}_1 = T_1\boldsymbol{d}_1 + \dfrac{\mathrm{d}\,W_0}{\mathrm{d}\,\sin\gamma}\boldsymbol{d}_2 + \Lambda\tau_1 k_1\boldsymbol{b}_1 - \Gamma\boldsymbol{d}_{1,xx} \\ \boldsymbol{F}_2 = T_2\boldsymbol{d}_2 + \dfrac{\mathrm{d}\,W_0}{\mathrm{d}\,\sin\gamma}\boldsymbol{d}_1 + \Lambda\tau_2 k_2\boldsymbol{b}_2 - \Gamma\boldsymbol{d}_{2,yy} \\ \Lambda\dfrac{\partial}{\partial x}(\tau_1) = 0 \\ \Lambda\dfrac{\partial}{\partial y}(\tau_2) = 0 \\ \dfrac{\partial\boldsymbol{F}_1}{\partial x} + \dfrac{\partial\boldsymbol{F}_2}{\partial y} + \boldsymbol{f} = \boldsymbol{0} \end{cases} \tag{21}$$

or equivalently

$$\begin{cases} \boldsymbol{F}_1 = (T_1 + \Gamma(k_1)^2)\boldsymbol{d}_1 + \dfrac{\mathrm{d}\,W_0}{\mathrm{d}\,\sin\gamma}\boldsymbol{d}_2 + (\Lambda - \Gamma)\tau_1 k_1\boldsymbol{b}_1 - \Gamma k_{1,x}\,\boldsymbol{n}_1 \\ \boldsymbol{F}_2 = (T_2 + \Gamma(k_2)^2)\boldsymbol{d}_2 + \dfrac{\mathrm{d}\,W_0}{\mathrm{d}\,\sin\gamma}\boldsymbol{d}_1 + (\Lambda - \Gamma)\tau_2 k_2\boldsymbol{b}_2 - \Gamma k_{2,y}\,\boldsymbol{n}_2 \\ \Lambda\dfrac{\partial}{\partial x}(\tau_1) = 0 \\ \Lambda\dfrac{\partial}{\partial y}(\tau_2) = 0 \\ \dfrac{\partial\boldsymbol{F}_1}{\partial x} + \dfrac{\partial\boldsymbol{F}_2}{\partial y} + \boldsymbol{f} = \boldsymbol{0}. \end{cases} \tag{22}$$

Equations (22) describe the mechanical behaviour of sets of inextensible fibers forming a surface when shearing, twisting and bending effects are taken into account, such that some elementary modes of the behaviours of woven fabric, where the fibers are the weft and the warp, can be exhibited.

5. Conclusions

An overwiev on the works of Wang and Pipkin [5] and of Luo and Steigmann [1] has been given. A first model that describes a fabric formed by inextensible fibers has been found. In this model, both shear and twist are considered. The balance equations are expressed through the torsions τ_1, τ_2 and the curvatures k_1, k_2 of the d_1, d_2 families of fibers. Strain energy for sets of fibers that undergo shear, twist and bending has been given in a suitable explicit form.

The results presented in this paper represent a first step of my research, whose main purpose is to develop a model for textile fabrics within the framework of Cosserat shell theory, where the shell itself is made of two families of Cosserat rods, namely the weft and the warp of the fabric. This model would encompass more refined phenomena, typical of fabrics, such as wrinkles and krinkles.

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