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A CHARACTERIZATION OF THE EXPONENTIAL STABILITY OF EVOLUTIONARY PROCESSES IN TERMS OF THE ADMISSIBILTY OF ORLICZ SPACES

Abstract. A characterization of the exponential stability of evolutionary rocesses in terms of the admissibility of some pairs of spaces, is given. The method of "test functions" from a very large class of spaces is used. Thus are obtained generalizations of some results given by N. van Minh, F. Rabiger and R. Schnaubelt.

1. Introduction

In this paper we establish that an evolutionary process $\mathcal{U} = \{U(t,s)\}_{t\geq s\geq 0}$ on a Banach space X is exponentially stable if (and only if) for each measurable X-valued function f from a Banach space $E(X)$ the function

(1) $$x_f(t) = \int_0^t U(t,\tau)f(\tau)d\tau, \qquad t \geq 0$$

lies in $E(X)$. The construction of $E(X)$ is related to an Orlicz space E (i.e $E(X)$ is the space of all X-valued measurable functions f, such that $t \mapsto \|f(t)\| : \mathbb{R}_+ \to \mathbb{R}_+$ is in E.) Our paper continues the line of research that characterizes exponential stability in terms of "Perron-type" theorems. For instance, consider a nonautonomous abstract Cauchy problem,

(2) $$x'(t) = A(t)x(t), \quad x(s) = x_s, \quad x_s \in D(A(s)) \quad t \geq s \geq 0$$

on a Banach space X. Assume, for a moment, that 2 is well-posed in the sense that there exists an evolutionary process $\{U(t,s)\}_{t\geq s\geq 0}$ which gives a differentiable solution $x(\cdot)$. This means that $x(\cdot) : t \mapsto U(t,s)x(s), t \geq s \geq 0$, is differentiable for any given initial conditions $x(s) = x_s \in D(A(s)), x(t) \in D(A(t))$, and 2 holds. Now let f be a locally integrable X - valued function on R_+ and consider the inhomogeneous Cauchy problem:

(3) $$x'(t) = A(t)x(t) + f(t), \qquad x(0) = 0, \qquad t \geq 0$$

The function u_f given by 1 is called a *mild* solution of 3. In some well-known particular cases (see for instance [9]) u_f is the unique classical solution of 3. This type of characterization of exponential stability for 2 has a fairly long history that goes back

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to the paper ”Die stabilitätsfrage bei differentialgeighungen” [14],where Perron gave a characterization of exponential stability of the solutions to the linear differential equations

$$\frac{dx}{dt} = A(t)x, \quad t \in [0, +\infty), \quad x \in \mathbb{R}^n$$

where $A(t)$ is a matrix bounded continuous function, in terms of the existence of bounded solutions of the equations

$$\frac{dx}{dt} = A(t)x + f(t)$$

where f is a continuous bounded function on $\mathbb{R}_+$. After these seminal researches of O. Perron, relevant results concerning the extension of Perron's problem in the more general framework of infinite-dimensional Banach spaces were obtained in pioneering monographs by M. G. Krein, J. L. Daleckij, R. Bellman, J. L. Massera and J. J. Schäffer. In last few years, several results about exponential stability and exponential dichotomy for the case of exponentially bounded and strongly continuous evolution families were obtained by N. van Minh ([11], [12]), F. Räbiger [11] , Y. Latushkin ([2], [6], [7], [8]), M. Megan ([10], [15]), P. Preda ([15], [16], [17]), T. Randolph ([7], [8]), R. Schnaubelt ([8], [11], [18]).

In the spirit of Perron's idea, the aim of this paper is to propose a very general and useful approach for the subject concerning the connection between the exponential stability of evolutionary processes and the admissibility of certain function spaces, using a well-known class of function spaces which are translation invariant and also rearrangement invariant(see for instance ([13]). Theses type of spaces are called in the literature Orlicz spaces. Related to the subject above, we note that until now the most common classes of spaces used as ”input” and ”output” spaces were the L^p spaces. These are in particular Orlicz spaces, so this treatment include as particular cases the situations above. A unified treatment can be imposed for the connection between the admissibility and exponential stability, since the present characterization includes as particular cases many interesting situations. Among them, we note (L^p, L^p)-admissibility. Also, our results generalize some well-known results due to N. van Minh, F. Rabiger and R. Schnaubelt. Moreover, the paper breaks new ground and this approach brings as particular cases other useful situations.

2. Preliminaries

In this section we list the principal notations and symbols. For a Banach space X we denote by $\mathcal{M}(\mathbb{R}_+, X)$ the space of all Bochner measurable functions from $\mathbb{R}_+$ to X and

$$L^1_{loc}(\mathbb{R}_+, X) = \left\{ f \in \mathcal{M}(\mathbb{R}_+, X) : \int_K \| f(t)) \| dt < \infty \right\}$$

for each compact $K \in \mathbb{R}_+$,

$$L^p(\mathbb{R}_+, X) = \left\{ f \in \mathcal{M}(\mathbb{R}_+, X) : \int_{\mathbb{R}_+} \| f(t) \|^p dt < \infty, \ p \in [1, \infty) \right\}$$

$$L^{\infty}(\mathbb{R}_+, X) = \left\{ f \in \mathcal{M}(\mathbb{R}_+, X) : \ ess \sup_{t\in\mathbb{R}_+} \| f(t)\| < \infty, \ p \in [1,\infty) \right\}$$

It is known that $L^p(\mathbb{R}_+, X)$, $L^{\infty}(\mathbb{R}_+, X)$ are Banach spaces endowed with the respectively norms:

$$||f||_p = \left(\int_{\mathbb{R}_+} ||f(t)||^p dt \right)^{\frac{1}{p}}$$

$$||f||_{\infty} = ess \sup_{t\in\mathbb{R}_+} ||f(t)||$$

In order to simplify the notations we put $L^p := L^p(\mathbb{R}_+, \mathbb{R})$, $L^{\infty} := L^{\infty}(\mathbb{R}_+, \mathbb{R})$, for all $p \in [1, \infty)$ and $L^1_{loc} = L^1_{loc}(\mathbb{R}_+, \mathbb{R})$.

Now, for the convenience of the reader, we recall the definition of Orlicz spaces.

Let $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ be a function that is non-decreasing, left-continuous and has the property $\varphi(t) > 0$, for all $t > 0$. Define

$$\Phi(t) = \int_0^t \varphi(s)ds$$

A function Φ of this form is called a *Young function*. For $f : \mathbb{R}_+ \to \mathbb{R}$ a measurable function and Φ a Young function we define

$$M^{\Phi}(f) = \int_0^{\infty} \Phi(|f(s)|)ds.$$

The set L^{Φ} of all f for which there exists a $k > 0$ such that $M^{\Phi}(kf) < \infty$ is easily checked to be a linear space. With the norm

$$\| f\|_{\Phi} = \inf\left\{ k > 0 : M^{\Phi}(\frac{1}{k} f) \leq 1 \right\}$$

the space $(L^{\Phi}, \| \cdot \|_{\Phi})$ becomes a special type of Banach space called an Orlicz space.

REMARK 1. It is easy to check that $\chi_{[0,t]} \in L^{\Phi}$ and

$$\|\chi_{[0,t]}\|_{\Phi} = \frac{1}{\Phi^{-1}\left(\frac{1}{t}\right)}$$

for all $t > 0$.

EXAMPLE 1. The L^p spaces are examples of Orlicz spaces

The connection between Orlicz spaces and the L^p spaces is given by

REMARK 2. $L^{\Phi} = L^p$ if and only if $\Phi(t) = t^p$, for all $t \geq 0$.

The "only if" part is obvious. Conversely if $L^\Phi = L^p$ then $\|\chi_{[0,t]}\|_\Phi = \|\chi_{[0,t]}\|_p$, for all $t > 0$, and so $\Phi^{-1}(s) = s^{\frac{1}{p}}$ for all $s > 0$, which implies that $\Phi(t) = t^p$, for all $t \geq 0$.

EXAMPLE 2. If we take $\Phi(t) = e^t - 1$ then $L^\Phi \subset L^p$, for all $p \in [1, \infty)$.

Indeed one can see that $t^m \leq m!\Phi(t)$ for all $t \geq 0$ and all $m \in \mathbb{N}^*$ which implies that $L^\Phi \subset L^m$, for all $m \in \mathbb{N}^*$. Having in mind that $L^m \bigcap L^{m+1} \subset L^p$ for all $p \in [m, m+1]$, and all $m \in \mathbb{N}^*$, it follows that $L^\Phi \subset L^p$, for all $p \in [m, m+1]$, and all $m \in \mathbb{N}^*$.

In what follows, if E is a Orlicz space then we denote by

$$E(X) = \{f \in \mathcal{M}(\mathbb{R}_+, X) : t \mapsto \|f(t)\| : \mathbb{R}_+ \to \mathbb{R} \text{ is in } E\}.$$

REMARK 3. $E(X)$ is a Banach space endowed with the norm

$$\|f\|_{E(X)} = \| \, \|f(\cdot)\| \, \|_E$$

REMARK 4. If $\{f_n\}_{n\in\mathbb{N}} \subset E(X)$, $f \in E(X)$, and $f_n \to f$ in $E(X)$ when $n \to \infty$, then there exists $\{f_{n_k}\}_{k\in\mathbb{N}}$ a subsequence of $\{f_n\}_{n\in\mathbb{N}}$ such that

$$f_{n_k} \to f \ a.e.$$

For proof of this see [19].

PROPOSITION 1. *If Φ is a Young function of the Orlicz space L^Φ then*

i) The map $a_\Phi : \mathbb{R}_+^* \to \mathbb{R}_+^*$ given by $a_\Phi(t) = t\Phi^{-1}\left(\frac{1}{t}\right)$ is nondecreasing.

ii) $\int\limits_0^t |f(s)|ds \leq a_\Phi(t)\|f\|_\Phi$, for all $t > 0$, $f \in L^\Phi$.

Proof. i) First let us prove that the map $b : \mathbb{R}_+^* \to \mathbb{R}_+^*$, given by $b(u) = \frac{\Phi(u)}{u}$ is nondecreasing. If $0 < u_1 \leq u_2$ then

$$\frac{\Phi(u_1)}{u_1} = \frac{1}{u_1}\int\limits_0^{u_1} \varphi(s)ds = \frac{1}{u_1}\int\limits_0^{u_2} \varphi\left(\frac{u_1}{u_2}v\right)\frac{u_1}{u_2}dv =$$

$$= \frac{1}{u_2}\int\limits_0^{u_2} \varphi\left(\frac{u_1}{u_2}v\right)dv \leq \frac{1}{u_2}\int\limits_0^{u_2} \varphi(v)dv = \frac{\Phi(u_2)}{u_2}.$$

In order to prove that a_Φ is nondecreasing let $0 < t_1 \leq t_2$. It follows that $0 < w_2 := \Phi^{-1}\left(\frac{1}{t_2}\right) \leq \Phi^{-1}\left(\frac{1}{t_1}\right) := w_1$ and so $b(w_2) \leq b(w_1)$. Having in mind that

$$b(w_1) = \frac{1}{a_\Phi(t_1)} \quad \text{and} \quad b(w_2) = \frac{1}{a_\Phi(t_2)},$$

it follows that a_Φ is a nondecreasing function.

ii) Consider $f \in L^\Phi, t > 0, k > 0$ such that $M^\Phi\left(\frac{1}{k}f\right) \leq 1$. Then we have that

$$\Phi\left(\frac{1}{kt}\int_0^t |f(s)|ds\right) \leq \frac{1}{t}\int_0^t \Phi\left(\frac{1}{k}|f(s)|\right)ds \leq \frac{1}{t},$$

and so

$$\int_0^t |f(s)|ds \leq t\Phi^{-1}\left(\frac{1}{t}\right)k,$$

which implies that

$$\int_0^t |f(s)|ds \leq t\Phi^{-1}\left(\frac{1}{t}\right)\|f\|_\Phi = a_\Phi(t)\|f\|_\Phi$$

for all $t > 0,\ f \in L^\Phi$. □

REMARK 5. Using a simple translation argument we may state that

$$\int_{t_0}^{t_0+t} |f(s)|ds \leq a_\Phi(t)||f||_\Phi$$

for all $t_0 \geq 0, t > 0,\ f \in L^\Phi$.

DEFINITION 1. *A family of bounded linear operators acting on X and denoted by $\mathcal{U} = \{U(t,s)\}_{t\geq s\geq 0}$ is called an evolutionary process if the following statements hold:*

e_1) $U(t,t) = I$ *(where I is the identity operator on X), for all $t \geq 0$;*

e_2) $U(\cdot,s)x$ *is continuous on* $[s,\infty)$*, for all $s \geq 0,\ x \in X$;* $U(t,\cdot)x$ *is continuous on* $[0,t]$*, for all $t \geq 0,\ x \in X$;*

e_3) $U(t,s) = U(t,r)U(r,s)$*, for all $t \geq r \geq s \geq 0$;*

e_4) *there exist $M, \omega > 0$ such that*

$$\|U(t,s)\| \leq Me^{\omega(t-s)} \quad \text{for all} \quad t \geq s \geq 0.$$

DEFINITION 2. *The evolutionary process $\mathcal{U} = \{U(t,s)\}_{t\geq s\geq 0}$ is called uniformly exponentially stable (u.e.s) if there exist two strictly positive constants N, ν such that the following statement hold:*

$$\|U(t,s)\| \leq Ne^{-\nu(t-s)}.$$

DEFINITION 3. *The pair (E,F) of Orlicz spaces is said to be admissible to the evolutionary process $\mathcal{U}$ if for all $f \in E(X)$ the function $x_f : \mathbb{R}_+ \to X$ defined by $x_f(t) = \int_0^t U(t,s)f(s)ds$ lies in $F(X)$.*

3. The main result

LEMMA 1. *If the pair (E, F) of Orlicz spaces is admissible to $\mathcal{U}$ then there exists $K > 0$ such that*

$$\|x_f\|_{F(X)} \leq K\|f\|_{E(X)}.$$

Proof. We define the linear operator $T : E(X) \to F(X)$ given by

$$(Tf)(t) = \int_0^t U(t,s)f(s)ds.$$

If $\{g_n\}_{n\in\mathbb{N}} \subset E(X)$, $g \in E(X)$, and $h \in F(X)$ are such that

$$g_n \stackrel{E(X)}{\longrightarrow} g, \quad Tg_n \stackrel{F(X)}{\longrightarrow} h,$$

then

$$\|(Tg_n)(t) - (Tg)(t)\| \leq \int_0^t \|U(t,s)(g_n(s) - g(s))\|ds \leq$$
$$\leq \int_0^t Me^{\omega t}\|g_n(s) - g(s)\|ds$$
$$\leq Me^{\omega t}a_\Phi(t)\|g_n - g\|_{E(X)},$$

for all $t \geq 0$ and all $n \in \mathbb{N}$. It follows, using again the Remark 2, that $Tg = h$, and hence T is closed and so, by the Closed-Graph Theorem it is also bounded. So we obtain that

$$\|x_f\|_{F(x)} = \|Tf\|_{F(X)} \leq \|T\| \, \|f\|_{E(X)}, \text{ for all } f \in E(X) \quad \text{as required.}$$

□

LEMMA 2. *If L^Φ is an Orlicz space, $h \in L^\Phi$, $h \geq 0$ and there are two positive constants a and b such that $h(r) \leq ah(t) + b$, for all $r \geq t \geq 0$ with $r - t \leq 1$ then $h \in L^\infty$.*

Proof. By the hypothesis we have that

$$h(n+1) \leq ah(s) + b, \text{ for all } n \in \mathbb{N} \text{ and all } s \in [n, n+1]$$

and from here

$$h(n+1) \leq a\int_n^{n+1} h(s)ds + b \leq aa_\Phi(1)\|h\|_{L^\Psi} + b, \text{ for all } n \in \mathbb{N},$$

which implies that

$$c = \sup_{n\in\mathbb{N}} h(n) < \infty.$$

Using again the hypothesis, we obtain that

$$h(t) \leq ah(n) + b \leq ac + b, \text{ for all } \ n \in \mathbb{N}, \text{ and all } t \in [n, n+1].$$

□

LEMMA 3. *If the pair (L^{Φ}, L^{Φ}) is admissible to $\mathcal{U}$, where Φ is a Young function, then the following statements hold:*

i) For all $f \in L^{\Phi}(X)$ there exist $a, b > 0$ such that

$$\|x_f(r)\| \leq a\|x_f(t)\| + b, \text{ for all } r \geq t \geq 0 \text{ with } r - t \leq 1;$$

ii) the pair (L^{Φ}, L^{∞}) is admissible to $\mathcal{U}$.

Proof. i) We have that

$$\begin{aligned} x_f(r) &= \int_0^r U(r,s)f(s)ds = \\ &= \int_0^t U(r,t)U(t,s)f(s)ds + \int_t^r U(r,s)f(s)ds \\ &= U(r,t)x_f(t) + \int_t^r U(r,s)f(s)ds, \text{ for all } r \geq t \geq 0. \end{aligned}$$

Therefore

$$\begin{aligned} \|x_f(r)\| &\leq Me^{\omega(r-t)}\|x_f(t)\| + \int_t^r Me^{\omega(r-s)}\|f(s)\|ds \\ &\leq Me^{\omega}\|x_f(t)\| + Me^{\omega}\int_t^{t+1}\|f(s)\|ds \\ &\leq Me^{\omega}\|x_f(t)\| + Me^{\omega}a_{\Phi}(1)\|f\|_{L^{\Phi}(X)} \end{aligned}$$

for all $r \geq t \geq 0$ with $r - t \leq 1$.
The condition ii) follows directly from i) and Lemma 2. □

LEMMA 4. *Let $g : \{(t, t_0) \in \mathbb{R}^2 : t \geq t_0 \geq 0\} \to \mathbb{R}_+$ be a function such that the following properties hold:*

1) $g(t, t_0) \leq g(t, s)g(s, t_0)$, for all $t \geq s \geq t_0 \geq 0$;

2) there exist $M, a > 0$ and $b \in (0, 1)$ such that

$$g(t, t_0) \leq M, \quad \text{for all } t_0 \geq 0 \text{ and all } t \in [t_0, t_0 + a],$$

$$g(t_0 + a, t_0) \leq b, \quad \text{for all } t_0 \geq 0.$$

Then there exist two constants $N, \nu > 0$ such that

$$g(t, t_0) \leq Ne^{-\nu(t-t_0)}, \text{ for all } t \geq t_0 \geq 0.$$

Proof. Let $t \geq t_0 \geq 0$ and $n = \left[\frac{t-t_0}{a}\right]$, the largest integer less or equal than $\frac{t-t_0}{a}$. Then we have that

$$g(t,t_0) \leq g(t,t_0+na)g(t_0+na,t_0) \leq$$
$$\leq g(t,t_0+na)b^n \leq Mb^n = Me^{-\nu na} \leq Ne^{-\nu(t-t_0)}$$

where $\nu = -\frac{1}{a}\ln b,\ N = Me^{\nu a}$, as required. □

Now we can state the main result of this paper.

THEOREM 1. *$\mathcal{U}$ is u.e.s. if and only if there exists an Orlicz space L^Φ such that the pair (L^Φ, L^Φ) is admissible to $\mathcal{U}$.*

Proof. Necessity. It follows easily from Definition 2 that the pair (L^∞, L^∞) is admissible to $\mathcal{U}$.

Sufficiency. First observe that if the pair (L^Φ, L^Φ) is admissible to $\mathcal{U}$ then by Lemma 3 the pair (L^Φ, L^∞) is admissible to $\mathcal{U}$. Let $x \in X,\ t_0 \geq 0$ and $f : \mathbb{R}_+ \to X$,

$$f(t) = \begin{cases} U(t,t_0)x, & t \in [t_0, t_0+1] \\ 0, & t \in \mathbb{R}_+ \setminus [t_0, t_0+1] \end{cases}$$

It is easy to check that $f \in L^\Phi(X)$ and $\|f\|_{L^\Phi(X)} \leq Me^{\omega}\frac{1}{\Phi^{-1}(1)}\|x\|$ and

$$x_f(t) = \begin{cases} 0, & 0 \leq t \leq t_0 \\ \\ \int_{t_0}^{t_0+1} U(t,s)f(s)ds, & t \geq t_0+1 \end{cases}$$

If $t \geq t_0 + 1$ then

$$x_f(t) = \int_{t_0}^{t_0+1} U(t,s)U(s,t_0)x\,ds = U(t,t_0)x$$

which implies that

$$\|U(t,t_0)x\| = \|x_f(t)\| \leq \|x_f\|_\infty \leq K\|f\|_{L^\Phi(X)} \leq KMe^{\omega}\frac{1}{\Phi^{-1}(1)}\|x\|$$

for all $t \geq t_0 + 1,\ t_0 \geq 0$ and all $x \in X$. Hence there exits $L > 0$ such that

$$||U(t,t_0)|| \leq L, \text{ for all } t \geq t_0 \geq 0.$$

Let $t_0 \geq 0,\ \delta > 0,\ x \in X$ and $g : \mathbb{R}_+ \to X$

$$g(t) = \begin{cases} U(t,t_0)x, & t \in [t_0, t_0+\delta] \\ 0, & t \in \mathbb{R}_+ \setminus [t_0, t_0+\delta] \end{cases}$$

Then $g \in L^\Phi(X)$, and $\|g\|_{L^\Phi(X)} \leq L\frac{1}{\Phi^{-1}\left(\frac{1}{\delta}\right)}\|x\|$. It follows that

$$x_g(t) = \int_0^t U(t,s)f(s)ds = \begin{cases} 0, & t \in [0,t_0) \\ (t-t_0)U(t,t_0)x, & t \in [t_0, t_0+\delta) \\ \delta U(t,t_0)x, & t \in [t_0+\delta, \infty) \end{cases}$$

and so

$$\frac{\delta^2}{2}\|U(t_0+\delta,t_0)x\| = \int_{t_0}^{t_0+\delta}(s-t_0)\|U(t_0+\delta,t_0)x\|ds \le \int_{t_0}^{t_0+\delta}(s-t_0)L\|U(s,t_0)x\|ds$$

$$= L\int_{t_0}^{t_0+\delta}\|x_g(s)\|ds \le La_\Phi(\delta)\|x_g\|_{L^\Phi(X)} \le KLa_\Phi(\delta)\|g\|_{E(X)} \le$$

$$\le KL^2 a_\Phi(\delta)\frac{1}{\Phi^{-1}(\frac{1}{\delta})}\|x\| \le KL^2\delta\|x\|$$

for all $t_0 \ge 0,\ \delta > 0,\ x \in X$. We obtain that

$$\|U(t_0+\delta,t_0)\| \le \frac{2KL^2}{\delta},\ \text{for all } t_0 \ge 0,\ \delta > 0.$$

By Lemma 4 it follows that there exist two constants $N, \nu > 0$ such that

$$||U(t,t_0)|| \le Ne^{-\nu(t-t_0)},\ \text{for all } t \ge t_0 \ge 0.$$

□

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