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ON CURVES ON RATIONAL NORMAL SCROLL SURFACES

Abstract. We extend the study of the Hartshorne - Rao module begun in [2] and [3] to any curve C on any rational normal scroll surface. We calculate the Rao function, the index of speciality and the Hilbert function of C; we characterize the arithmetically Cohen-Macaulay curves and we show that on a rational normal scroll surface the only arithmetically Gorenstein-curves are the trivial ones, i.e. the lines and the twisted anticanonical divisors.

Introduction

In this paper, we continue the study of the Rao module of curves on rational normal scroll begun in [2] and [3], by considering the more general ruled surfaces embedded in the projective space $\mathbb{P}^{e+2n+1}$ via the linear system $|C_0 + (e+n)\mathfrak{f}|$, where $e \geq 0$, $n \geq 1$, C_0 is a rational normal curve of degree n and self-intersection $-e$ and $\mathfrak{f}$ is a line such that $\mathfrak{f}^2 = 0$. We denote these scrolls by $S_{e,n}$. These smooth surfaces are arithmetically Cohen-Macaulay and they have degree $e + 2n$, so our interest was born because $S_{e,n}$ are "almost all" surfaces of minimal degree, in fact a classical result ensures that they are all the minimal surfaces different from the plane in $\mathbb{P}^2$ and the Veronese surfaces in $\mathbb{P}^5$.

Some results of this paper appear in [4].

We consider only smooth rational normal scroll surfaces, because the study on the Hartshorne-Rao module of a curve on a singular surface of minimal degree is trivial, as R. Ferraro proved in [7, Example 5.2].

In Section 1, we give the explicit calculation of the Rao and the Hilbert functions, the postulation and other cohomological information on any curve lying on a smooth rational normal scroll surface.

In Section 2, we find again the arithmetically Cohen-Macaulay curves (cf. [10], [11]) and among them we pick up the arithmetically Gorenstein curves.

1. Numerical information on a curve on $S_{e,n}$

1.1. Notation

We follow the standard notation and definitions as in [6] and [8], $\mathbb{P}^N$ will denote the N-dimensional projective space $\mathbb{P}^N_k$ over an algebraically closed field k; $R = k[x_0, \dots, x_N]$ will be the graded ring of polynomials in the $N+1$ variables $x_0, \dots, x_N$ with coefficient in k.

$X \subset \mathbb{P}^N$ will be a closed subscheme; $\mathscr{O}_X$ denotes the sheaf of regular functions on X, $\mathscr{I}_X$ the ideal sheaf of X and ω_X the canonical sheaf of X. $H_X(j)$ and $P_X(j)$ will be respectively the Hilbert function and the Hilbert polynomial of X.

If $\mathscr{F}$ is a sheaf of modules over X, for $i \geq 0$, $H^i(X, \mathscr{F})$ denotes the $i-$th cohomology module of $\mathscr{F}$ and $H^i_*(X, \mathscr{F}) := \bigoplus_{j \in \mathbb{Z}} H^i(X, \mathscr{F}(j))$.

The dimension of such modules as k-vector spaces will be denoted by $h^i(X, \mathscr{F}(j))$. If there is no ambiguity on X, we simply write $H^i(\mathscr{F})$ and $h^i(\mathscr{F})$.

By *curve* we always mean a locally Cohen-Macaulay and equidimensional subscheme of dimension 1.

Finally, by $\lfloor \cdot \rfloor$ and $\lceil \cdot \rceil$ we will denote respectively the smallest integer less or equal and the largest integer greater or equal to the number in the bracket.

$S := S_{e,n} \subset \mathbb{P}^{e+2n+1}$ will be a rational normal scroll surface, i.e. a rational ruled surface with invariant e embedded via the very ample linear system $H \sim C_0 + (e+n)\mathfrak{f}$, where C_0 is the minimal directrix and $\mathfrak{f}$ is a generic fiber. We recall that in such a way, S is an arithmetically Cohen-Macaulay surface of degree $e + 2n$ and so it is a *surface of minimal degree*. Any divisor on S is linearly equivalent to $aC_0 + b\mathfrak{f}$ for some $a, b \in \mathbb{Z}$ and it is effective, so it is a curve, if $a, b \geq 0$ and they are not both 0. By using the simple rules of intersection and the adjunction formula, we get that if $C \sim aC_0 + b\mathfrak{f}$, then the degree of C is $an + b$ and the arithmetic genus is $p_C = (a-1)\left(b - 1 - \frac{1}{2}ae\right)$. In other words the Hilbert polynomial of C is $P_C(j) = j(an+b) - (a-1)\left(b - 1 - \frac{1}{2}ae\right) + 1$.

1.2. Rao function

The calculation of the Rao function of a curve follows the same arguments used in [2] for curves on $S_{e,1}$.

THEOREM 1. *The Rao function of $C \sim aC_0 + b\mathfrak{f}$ is the following*

1. *If* $j \leq \min\left\{\left\lfloor \frac{b-ae+e-2}{n} \right\rfloor, a-2, \left\lfloor \frac{b-(e+2)}{e+n} \right\rfloor\right\}$,
$$h^1(\mathscr{I}_C(j)) = 0;$$
2. *If* $\left\lfloor \frac{b-ae+e-2}{n} \right\rfloor < j \leq \min\left\{a-2, \left\lfloor \frac{b-(e+2)}{e+n} \right\rfloor\right\}$ *and* $\alpha := \left\lfloor \frac{b-2-jn}{e} \right\rfloor$,
$$h^1(\mathscr{I}_C(j)) = (a-\alpha-1)\left(\frac{e}{2}(a+\alpha) - b + jn + 1\right);$$
3. *If* $\min\left\{a-2, \left\lfloor \frac{b-(e+2)}{e+n} \right\rfloor\right\} < j < \max\left\{a, \left\lceil \frac{b}{e+n} \right\rceil\right\}$,
$$h^1(\mathscr{I}_C(j)) = j(na+b) - p_C + 1 - \frac{1}{2}(j+1)(j(e+2n)+2);$$
4. *If* $\max\left\{a, \left\lceil \frac{b}{e+n} \right\rceil\right\} \leq j < \left\lceil \frac{b-ae}{n} \right\rceil$ *and* $\alpha := \left\lfloor \frac{jn-b}{e} \right\rfloor$,
$$h^1(\mathscr{I}_C(j)) = (a+\alpha)\left(jn - b + 1 + \frac{e}{2}(a-\alpha-1)\right)$$

5. *If* $j \geq \max\left\{a, \left\lceil \frac{b}{e+n} \right\rceil, \left\lceil \frac{b-ae}{n} \right\rceil\right\}$,

$$h^1(\mathscr{I}_C(j)) = 0.$$

Proof. It is enough to argue as in Theorem 2.1 in [2]. □

REMARK 1. By examining the Rao function of a curve C in Theorem 1, we can obtain some useful consequences.

- First, we have to note that, also if in the Theorem 1 it seems that there are three different intervals where the Rao function can be non trivial, really for any curve at least one of the intervals 2. or 4. is empty, as we can verify with a simple calculation.

- Studying the Rao function, we get that in this case, it is connected, that is there is no gap.

- Moreover, we get that the value $j = a - 1$ is always in the interval 3. and by substitution we get
$$h^1(\mathscr{I}_C(a-1)) = 0,$$
so, by previous remark, the Rao function of a curve begins in degree a or ends in degree $a - 2$.

Following [9], we will use notation below.

NOTATION 1. Let C be a non-*aCM* curve, then we will denote

$$r_a(C) := \min\left\{j \in \mathbb{Z} | h^1(\mathscr{I}_C(j)) \neq 0\right\}, \tag{1a}$$

$$r_o(C) := \max\left\{j \in \mathbb{Z} | h^1(\mathscr{I}_C(j)) \neq 0\right\} \tag{1b}$$

and

$$\operatorname{diam}(C) := r_o(C) - r_a(C) + 1 \tag{1c}$$

The numbers $r_a(C)$ and $r_o(C)$ are just two of the many indices we can associate to a curve and they are among the first information we search on the Hartshorne-Rao module of a curve.

REMARK 2. Of course, now we can calculate both $r_a(C)$ and $r_o(C)$. But it needs many long and unpleasant calculations, involving many divisions.

If the invariant e is zero, then we get the following simpler result.

COROLLARY 1. *Let $C \sim aC_0 + b\mathfrak{f}$ be a curve on the rational normal scroll surface $S_{0,n}$. Then the non - zero values of the Rao function of C are the following:*

$$h^1(\mathscr{I}_C(j)) = j(an+b) - ab + a + b - (j+1)(nj+1) \tag{2}$$

in degree $r_a(C) \leq j \leq r_o(C)$, where

$$r_a(C) \begin{cases} = a & \text{if } b > (a-1)n \\ \geq \left\lfloor \frac{b-2}{n} \right\rfloor + 1 & \text{if } b \leq (a-1)n \end{cases}$$

$$r_o(C) \begin{cases} \leq \left\lceil \frac{b}{n} \right\rceil - 1 & \text{if } b > (a-1)n \\ = a-2 & \text{if } b \leq (a-1)n \end{cases}$$

Proof. It is enough to note that for $e = 0$ the intervals 2. and 4. in Theorem 1 are empty and $h^1(\mathscr{I}_C(a-1)) = 0$, as we note in Remark 1. □

Another particular case that is worth noting is that of the union of fibers.

COROLLARY 2. *Let $C \sim b\mathfrak{f} \subset S_{e,n}$ be an union of fibers. Then (recalling* (1)*)*

$$r_a(C) = 0 \qquad h^1(\mathscr{I}_C(0)) = -p_C = b - 1$$

and

$$r_o(C) = \begin{cases} \left\lceil \frac{b}{n} \right\rceil - 1 - \delta_{1,n} & \text{if } n \text{ divides } b \\ \left\lceil \frac{b}{n} \right\rceil - 1 - \delta_{1,r} & \text{if } n \text{ does not divide } b \\ & \text{and } r \text{ is the remainder of the division} \\ & \text{between } b \text{ and } n. \end{cases}$$

1.3. Hilbert function

Before calculating the Hilbert function we recall the following definition.

DEFINITION 1. *Let D be a divisor on a curve $C \subset \mathbb{P}^N$. The* index of speciality *of D is $\epsilon := \max\{j | h^1(\mathscr{O}_D(j)) \neq 0\}$.*

Again, we get the explicit calculation for curves on rational normal scroll surface.

PROPOSITION 1. *Let $C \sim aC_0 + b\mathfrak{f} \subset S$ be a curve. Then the index of speciality of C is*

$$\epsilon = \begin{cases} \min\left\{a-2, \left\lfloor \frac{b-(e+2)}{e+n} \right\rfloor\right\} & \text{if } b \neq a(e+n) - e - 2n + 2 \\ a-3 = \min\left\{a-2, \left\lfloor \frac{b-(e+2)}{e+n} \right\rfloor\right\} - 1 & \text{if } b = a(e+n) - e - 2n + 2. \end{cases}$$

Proof. By the sequence

$$0 \to \mathscr{O}_S(-C) \to \mathscr{O}_S \to \mathscr{O}_C \to 0, \tag{3}$$

and by Serre duality, we get that if $H^2(\mathscr{O}_S(jH - C)) = 0$ (i.e. if $K + C - jH$ is non-effective), then $h^1(\mathscr{O}_C(j)) = 0$. Otherwise,

$$h^1(\mathscr{O}_C(j)) = h^2(\mathscr{O}_S(jH - C)) - h^2(\mathscr{O}_S(j)) = \\ = h^0(\mathscr{O}_S(K + C - jH)) - h^0(\mathscr{O}_S(K - jH)).$$

So, $h^1(\mathscr{O}_C(j)) \neq 0$ if and only if $K + C - jH$ is an effective divisor and

$$h^0(\mathscr{O}_S(K + C - jH)) \neq h^0(\mathscr{O}_S(K - jH)). \tag{4}$$

By writing explicitly the coefficients, we easily see that:

- $K + C - jH$ is an effective divisor if and only if

  $$j \leq m := \min \left\{ a - 2, \left\lfloor \frac{b - (e + 2)}{e + n} \right\rfloor \right\}$$

  unless $b = (a - 2)(e + n) + e + 2$. In this case, $a - 2 = \left\lfloor \frac{b-(e+2)}{e+n} \right\rfloor$ and $a - 3$ is the maximum value of j such that $K + C - jH$ is effective.
- $K - jH$ is effective if and only if $j \leq -2$, unless $e = 0$, $n = 1$ and $j \leq -3$.

So $\epsilon = m$ (or $\epsilon = m - 1$ if $b = (a - 2)(e + n) + e + 2$) if and only if in this degree the inequality (4) holds.
If $m \geq -1$, then we get the thesis, since in (4) the first divisor is effective and the second one not.
If $m \leq -2$, with some calculations we get that $m = -2$ and the curve C has $a = 0$ or $b = 0, n = 1$. In both cases, by Riemann-Roch theorem on S, we get the inequality (4). This complete the proof. □

REMARK 3. During the proof we get for a curve $C \subset S_{e,n}$:

$$\epsilon \geq -2$$

in agreement with [12], and

$$\epsilon = -2 \quad \Leftrightarrow \quad \begin{array}{l} C \text{ is an union of fibers} \\ \text{or } C = C_0 \text{ and } n = 1. \end{array}$$

We note that to calculate the index of speciality of an union of distinct fibers, we could argue as follows, without recall Proposition 1. The index of speciality of a line is -2. Moreover, since we can consider C as a disjoint union of lines, recalling that for cohomology the distributive property with respect to the direct sums holds, the index of speciality of C is also -2.

Now, it is very easy to calculate the Hilbert function $H_C(j)$ and the postulation $h^0(\mathscr{I}_C(j))$ of any curve $C \subset S$.

THEOREM 2. *The Hilbert function of $C \sim aC_0 + b\mathfrak{f}$ is the following*

1. *If* $1 \leq j \leq \epsilon$,
$$H_C(j) = H_S(j) = \frac{1}{2}j^2 \deg S + \frac{1}{2}j(e+2) + 1;$$

2. *If* $j > \epsilon$,
$$H_C(j) = P_C(j) - h^1(\mathscr{I}_C(j)).$$

More precisely:

2.1 *If* $\epsilon \leq j < \max\left\{a, \left\lceil \frac{b}{e+n} \right\rceil\right\}$,
$$H_C(j) = \frac{1}{2}(j+1)[j(e+2n)+2];$$

2.2 *If* $\max\left\{a, \left\lceil \frac{b}{e+n} \right\rceil\right\} \leq j < \left\lceil \frac{b-ae}{n} \right\rceil$ *and* $\alpha := \left\lfloor \frac{jn-b}{e} \right\rfloor$,
$$H_C(j) = j \deg C - p_C + 1 - (a+\alpha)\left[jn - b + 1 + \frac{e}{2}(a-\alpha-1)\right]$$

2.3 *If* $j \geq \max\left\{a, \left\lceil \frac{b}{e+n} \right\rceil, \left\lceil \frac{b-ae}{n} \right\rceil\right\}$,
$$H_C(j) = P_C(j) = j \deg C - p_C + 1.$$

Proof. By the sequence
$$0 \to I_C \to R \to H^0_*(\mathscr{O}_C) \to H^1_*(\mathscr{I}_C) \to 0,$$
we get
$$H_C(j) = h^0(\mathscr{O}_C(j)) - h^1(\mathscr{I}_C(j)).$$
By Theorem 1, it is enough to calculate $h^0(\mathscr{O}_C(j))$. We know that
$$P_C(j) = h^0(\mathscr{O}_C(j)) - h^1(\mathscr{O}_C(j)).$$
If $j > \epsilon$, $h^0(\mathscr{O}_C(j)) = P_C(j)$ and the item 2. is completely proved by using Proposition 1.
If $1 \leq j \leq \epsilon$, the problem is that $h^1(\mathscr{O}_C(j)) \neq 0$, but we bypass it by arguing on the cohomology sequence of
$$0 \to \mathscr{O}_S(-C) \to \mathscr{O}_S \to \mathscr{O}_C \to 0$$
tensored by $\mathscr{O}_S(j)$. We get
$$0 \to H^0(\mathscr{O}_S(jH - C)) \to H^0(\mathscr{O}_S(j)) \to H^0(\mathscr{O}_C(j)) \to H^1(\mathscr{I}_C(j)) \to 0$$

and so $H_C(j) = h^0(\mathscr{O}_C(j)) - h^1(\mathscr{I}_C(j)) = h^0(\mathscr{O}_S(j)) - h^0(\mathscr{O}_S(jH - C))$. Since in the interval $1 \le j \le \epsilon$ the divisor $jH - C$ is non - effective, then we have to calculate just $h^0(\mathscr{O}_S(j))$ and this is very simple by Riemann-Roch Theorem.

$$\begin{aligned} h^0(\mathscr{O}_S(j)) &= \frac{1}{2}\ jH \cdot (jH - K) + 1 \\ &= \frac{1}{2} j^2 \deg S + \frac{1}{2} j(e+2) + 1, \end{aligned}$$

since $h^1(\mathscr{O}_S(j)) = 0$, for all j, $h^2(\mathscr{O}_S(j)) = h^0(\mathscr{O}_S(K - jH)) = 0$, for $1 \le j \le \max\left\{a, \left\lceil \frac{b}{e+n} \right\rceil\right\}$ and these values are in the interval of Item 1.. This complete the proof. □

By knowing the Hilbert function of any curve on S, it is very simple to calculate the postulation

$$h^0(\mathscr{I}_C(j)) = \dim R_j - H_C(j).$$

2. Some “arithmetical” properties

In this section we characterize the arithmetically Cohen-Macaulay curves on a smooth rational normal scroll surface and, among them, we find the arithmetically Gorenstein curves.

2.1. *aCM* curves on $S_{e,n}$

By knowing the Rao function of any curve by Theorem 1, it is immediate to find the arithmetically Cohen-Macaulay curves. In fact, it is enough to find when the Rao function is identically zero.
The following result has already been found by Miyazaki ([10, Lemma 2.12]) and Nagel ([11]), but we could give a different proof just by forcing the Rao function to vanish identically.

PROPOSITION 2. *A curve $C \sim aC_0 + b\mathfrak{f}$ on $S_{e,n}$ is arithmetically Cohen-Macaulay if and only if $(a-1)(e+n) - n + 1 \le b \le a(e+n) + 1$.*

REMARK 4. We note that, fixed $a \in \mathbb{N}$, we find $e + 2n$ values b such that the curve $aC_0 + b\mathfrak{f}$ is *aCM*. Is it an accident that $e + 2n$ is the dimension of the projective space where our surface “lives”?

REMARK 5. It is an open problem what immersion of a rational ruled surface X_e forces a curve to be *aCM*, and (even for a curve on a quadric!) what is the best embedding to get this condition.

2.2. *aG* curves on $S_{e,n}$

Now, among the *aCM* divisors on S, we are going to find the arithmetically Gorenstein curves.

THEOREM 3. *Let C be a curve on S. Then C is arithmetically Gorenstein if and only if C is linearly equivalent to a twisted anticanonical divisor (that is $C \sim tH - K$ for some $t \in \mathbb{Z}$) or C is a line.*

REMARK 6. The sufficient condition holds for any arithmetically Cohen-Macaulay curve on an arithmetically Cohen-Macaulay surface. The interest of the theorem is that on a rational normal scroll surface the twisted anticanonical divisors are the *only* arithmetically Gorenstein curves which can $G-$link two curves on S. This is very useful to study the liaison classes on a rational normal scroll surface (forthcoming paper). Moreover it is in agreement with some results in [1] and it is related whit the main result in [5], that is: on an *aCM* smooth surface there are at most finitely many complete linear systems, not of the type $|tH - K|$, containing integral arithmetically Gorenstein curves.

Before the proof, we state the following easy lemma.

LEMMA 1. *Let C be an arithmetically Gorenstein curve of degree d and genus g. Then $\omega_C \cong \mathcal{O}_C(t)$, where t is an integer such that*

$$td = 2g - 2$$

Theorem 3. Since C is a divisor on S, we get $K_C \sim (K + C)_{|C}$, so the hypothesis $\omega_C \cong \mathcal{O}_C(t)$ gets $(K + C - tH)_{|C}$ is equivalent to the trivial divisor. To prove that C is a twisted anticanonical divisor, we have to check $K + C - tH \sim 0$ as divisors on S. We begin arguing on the cohomology sequence of

$$0 \to \mathcal{O}_S(-C) \to \mathcal{O}_S \to \mathcal{O}_C \to 0$$

tensored by $\mathcal{O}_S(K + C - tH)$:

$$0 \to H^0(\mathcal{O}_S(K - tH)) \to H^0(\mathcal{O}_S(K + C - tH)) \to H^0(\mathcal{O}_C) \to 0.$$

The last term of the sequence is $H^1(\mathcal{O}_S(K - tH)) = H^1(\mathcal{O}_S(t)) = 0$, where the first equality holds by Serre duality and the second one because S is *aCM* .
Moreover, $h^0(\mathcal{O}_C) = 1$, as we can get by standard sequence

$$0 \to \mathcal{I}_C \to \mathcal{O}_{\mathbb{P}^N} \to \mathcal{O}_C \to 0,$$

since C is *aCM* . So,

$$h^0(\mathcal{O}_S(K + C - tH)) = 1 + h^0(\mathcal{O}_S(K - tH)).$$

By expliciting the coefficient, we get that the divisor $K - tH$ is non effective for $t \geq -1$, so

$$h^0(\mathcal{O}_S(K + C - tH)) = 1, \qquad \text{if } t \geq -1,$$

and this equality can hold only if $K + C - tH \sim \alpha C_0$.
We are going to show that $\alpha = 0$.
By contradiction, let $\alpha \neq 0$. Let $C \sim aC_0 + b\mathfrak{f}$, since $(K + C - tH)_{|C} \sim 0$, it is $\deg \alpha C_0 \cdot C = 0$ and so $b = ae$.
To show that this is a contradiction, we have to do some calculations. If $b = ae$, by previous Lemma 1, $t = \frac{ae-e-2}{e+n}$ and $\alpha = \frac{an-2n-e+2}{e+n}$.
Moreover, we recall that in particular C is *aCM* and so by Proposition 2, $an - 2n - e + 2 \leq 1$ and $\frac{1}{e+n} \geq \alpha \in \mathbb{N}$, so $\alpha = 0$. This is the contradiction.
We showed that the thesis holds if $t \geq -1$.
We have only to see what happens when $t \leq -2$. In this case $g + d - 1 \leq 0$, by Lemma 1. But, by using the *aCM* condition of Proposition 2, we get $g + d - 1 \geq \frac{1}{2}(e + 2n)(a - 1)$.
So, if $a \geq 2$, then it cannot be $t \leq -2$.
If $a = 1$, then $C \sim C_0 + b\mathfrak{f}$ and, since C is *aG*, $-2 = \deg K_C = t \deg C = t(n + b)$. This is possible with $t \leq -2$ if and only if $n = 1$ and $b = 0$, that is $C \sim C_0$ is a line on $S_{e,1}$.
If $a = 0$, then C is *aG* (so it is *aCM*) if and only if $C \sim \mathfrak{f}$. □

REMARK 7. We note that any line on S is not a twisted anticanonical divisor. In fact, a fiber $\mathfrak{f}$ could be in $|jH - K|$ if and only if $e + 2n = 1$ and it is impossible. Moreover C_0 is a twisted anticanonical divisor if $n = 2$ and in this case it is not a line.

We conclude by studying the self-linked curves on S.

COROLLARY 3. *If e is odd, then there is no self-linked curve on $S_{e,n}$; if e is even, then $C \sim aC_0 + b\mathfrak{f}$ is self-linked if and only if $b = a(e + n) - \frac{1}{2}(e + 2n) + 1$.*

Proof. By Theorem 3, it is enough to impose that $2C = tH - K$. □

REMARK 8. By Proposition 2, we can note that the self-linked curves on a scroll of even invariant e are *aCM* .

Finally, a question arises: we found the curves which are self-linked in their liaison class *on* the scroll, but there are some curves on S which are self-linked outside the scroll?

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